Spatial Analysis

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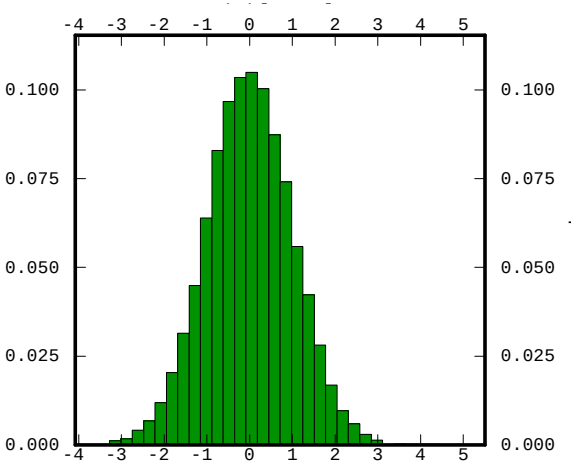
o Basic statistics

- Samples $\{z_i\}_{i=1,n}$
- Arithmetic mean $m = \frac{1}{n}\sum_{i=1}^{n} z_i$
- Variance $\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}(z_i - m)^2$
- Standard deviation σ
- Coefficient of variation $C_v = \frac{\sigma}{m}$

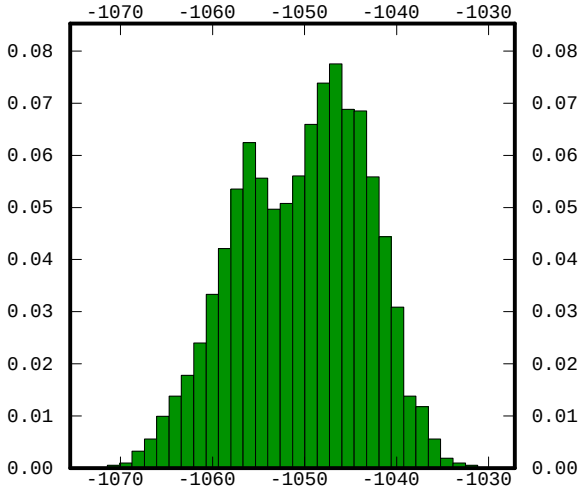
o Histograms

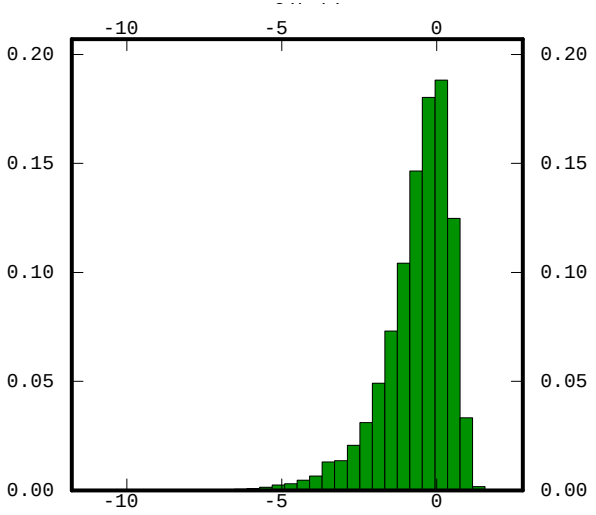
- Mode: most probable value
- Quantiles



Symmetrical

Multi-modal

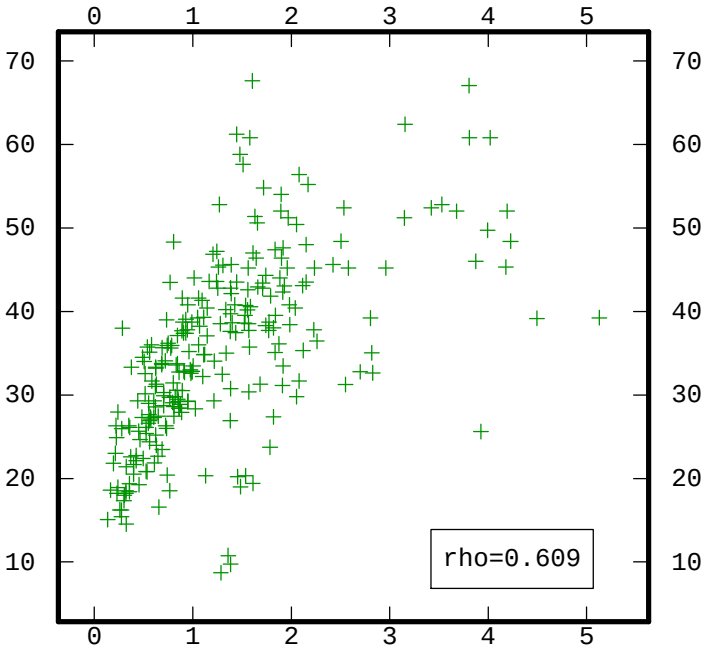




Asymmetrical

o Multivariate

- Variables $\left\{ z_i^1, z_i^2 \right\}_{i=1,n}$
- Covariance $C_{12} = \frac{1}{n} \sum_{i=1}^{n} \left(z_i^1 - m_1 \right) \left(z_i^2 - m_2 \right)$
- Correlation coefficient $\rho = \frac{C_{12}}{\sigma_1 \sigma_2}$
- Scatter plot



o Experimental variograms

- Data is a regionalized variable

$$z_i = z(x_i)$$

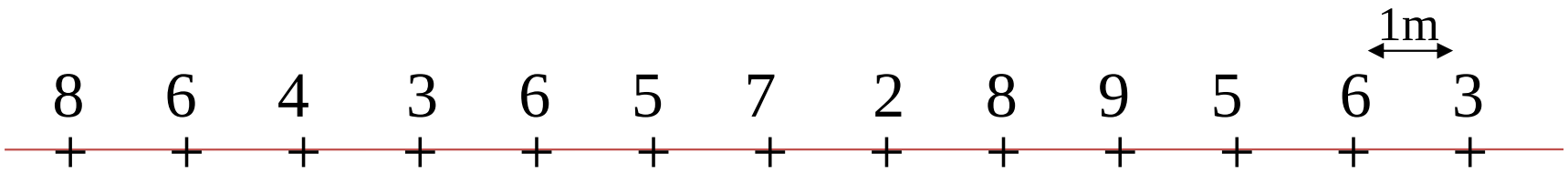
- The experimental variogram is a (discrete) function

$$\gamma(h) = \frac{1}{2N(h)} \sum_{i=1}^{N(h)} \left[z(x_i + h) - z(x_i) \right]^2$$

where N(h) is the number of pairs of points distant by h

o Experimental variogram for regular 1-D grid

- The variable is defined on a regular 1-D grid (mesh=1m)



- Calculate:
  - The mean and variance
  - The experimental variogram for the lag 1m, 2m and 3m

- What happens when replacing z by y, such that:

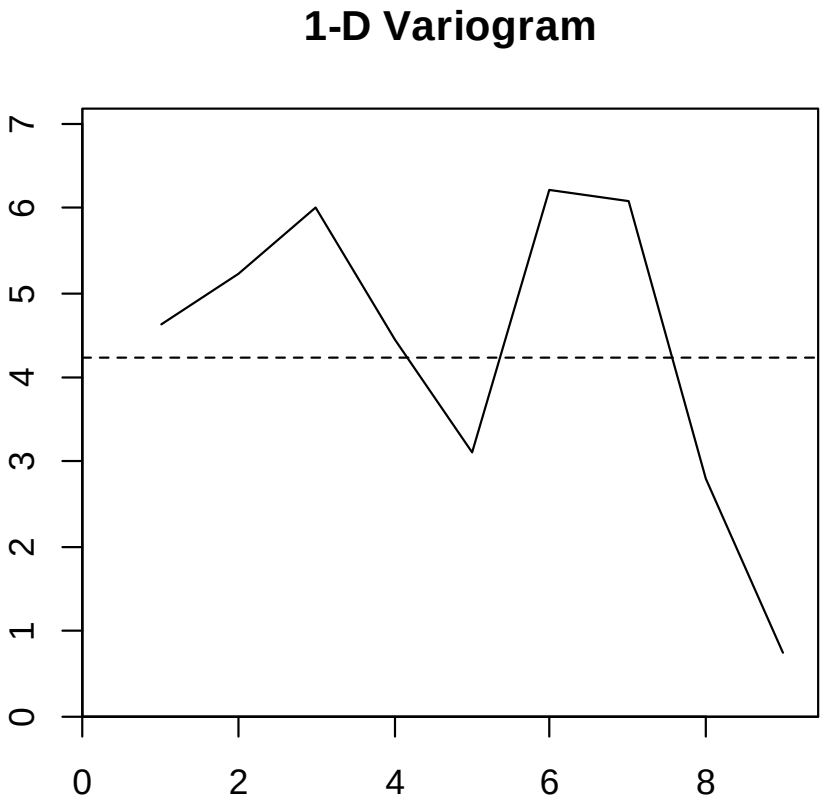
$$y(x) = z(x) + 3.2$$

o Experimental variogram for regular 1-D grid

➢ Results of the calculations

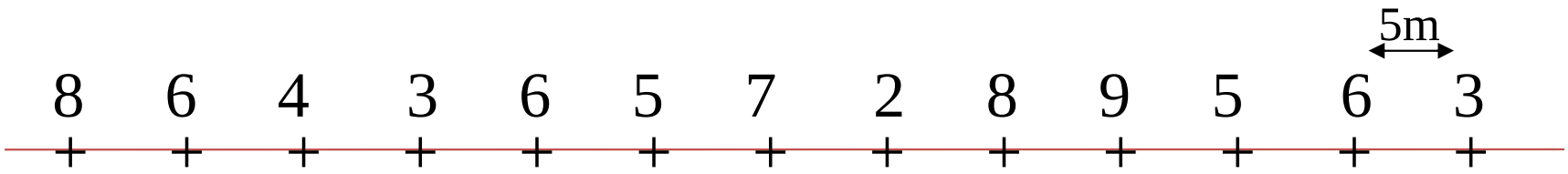
Referenced value (variance,...) = 4.249

Rank	Npairs	Distance	Value
1	12.000	1.000	4.625
2	11.000	2.000	5.227
3	10.000	3.000	6.000
4	9.000	4.000	4.444
5	8.000	5.000	3.125
6	7.000	6.000	6.214
7	6.000	7.000	6.083
8	5.000	8.000	2.800
9	4.000	9.000	0.750

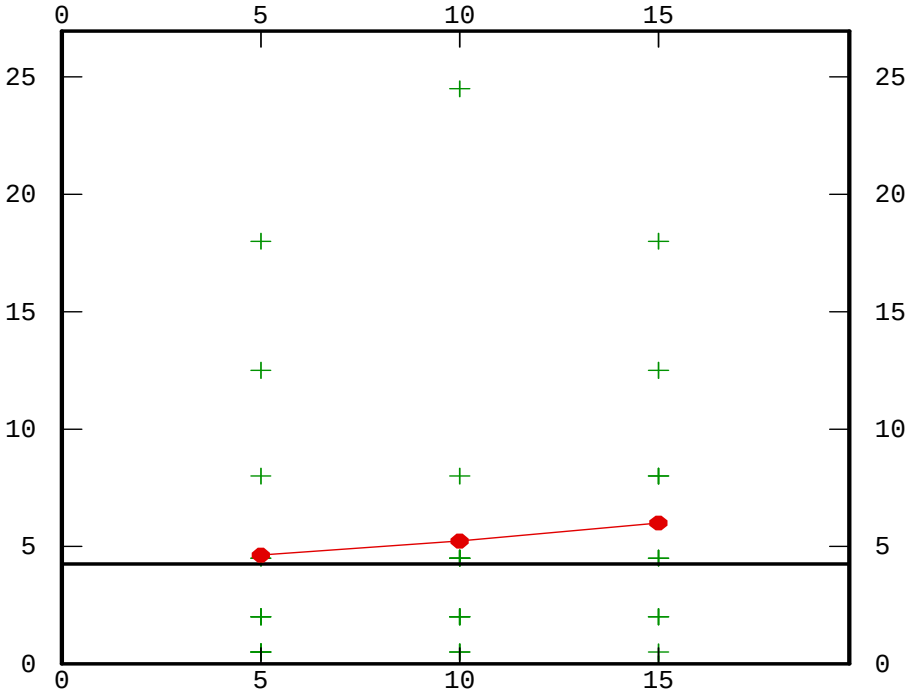


o Variogram cloud

- The variable is defined on a regular 1-D grid (mesh=5m)



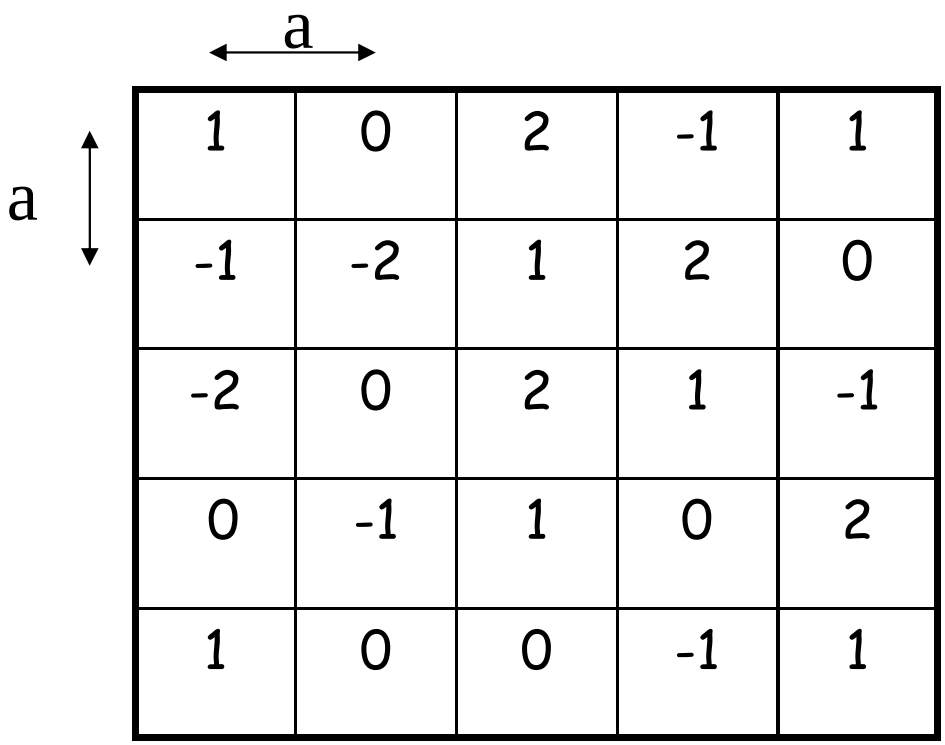
$$\frac{1}{2}(z_1 - z_2)^2$$



$$h(x_1, x_2)$$

o Experimental variogram for regular 2-D grid

- The variable is defined on a regular 2-D grid (square mesh=a)



1	0	2	-1	1
-1	-2	1	2	0
-2	0	2	1	-1
0	-1	1	0	2
1	0	0	-1	1

o Experimental variogram for regular 2-D grid

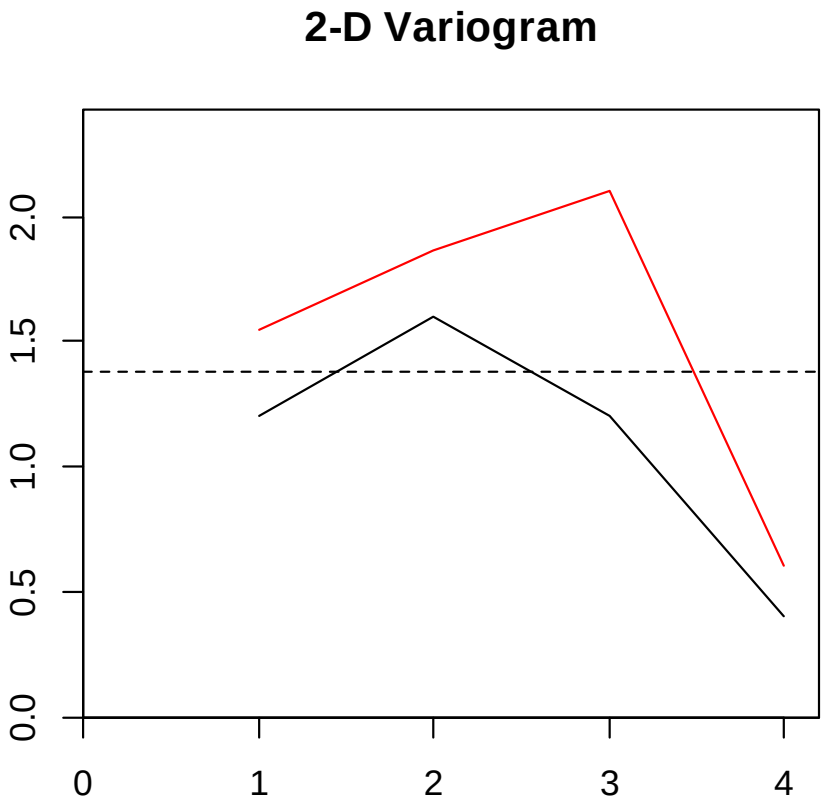
➢ Results of the calculations

Direction 1 (1.000 0.000)

Rank	Npairs	Distance	Value
1	20.000	1.000	1.200
2	15.000	2.000	1.600
3	10.000	3.000	1.200
4	5.000	4.000	0.400

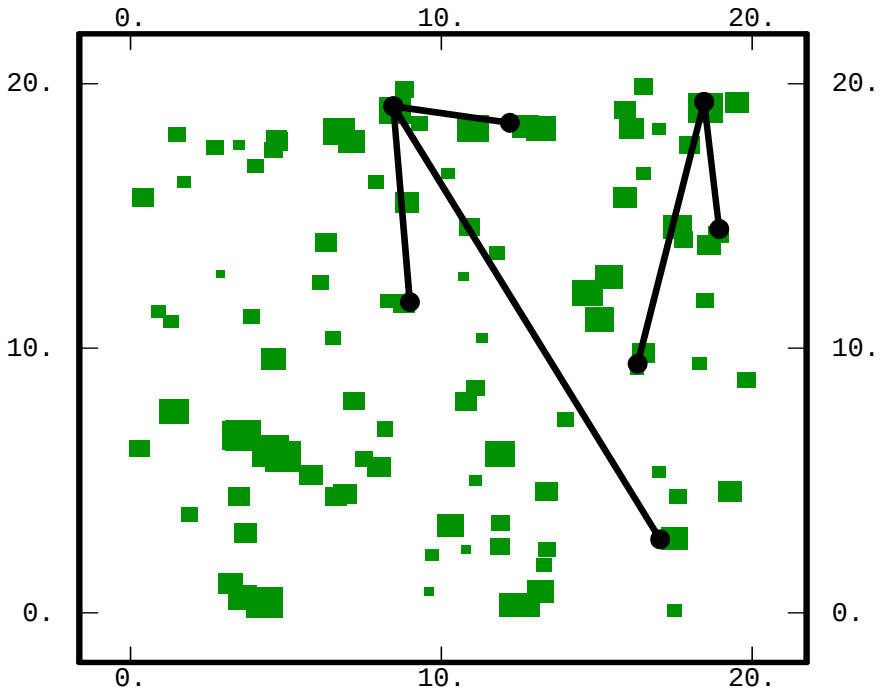
Direction 2 (0.000 1.000)

Rank	Npairs	Distance	Value
1	20.000	1.000	1.550
2	15.000	2.000	1.867
3	10.000	3.000	2.100
4	5.000	4.000	0.600

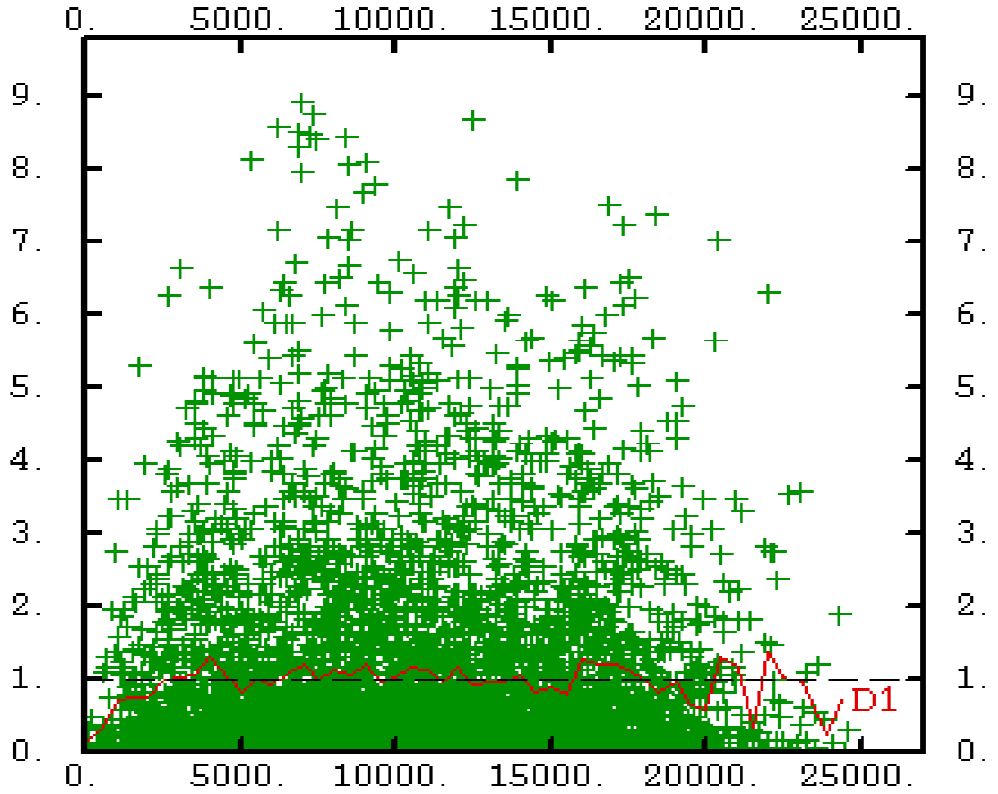


o Experimental variogram for irregular 2-D data set

➢ Calculation of the experimental omni-directional variogram



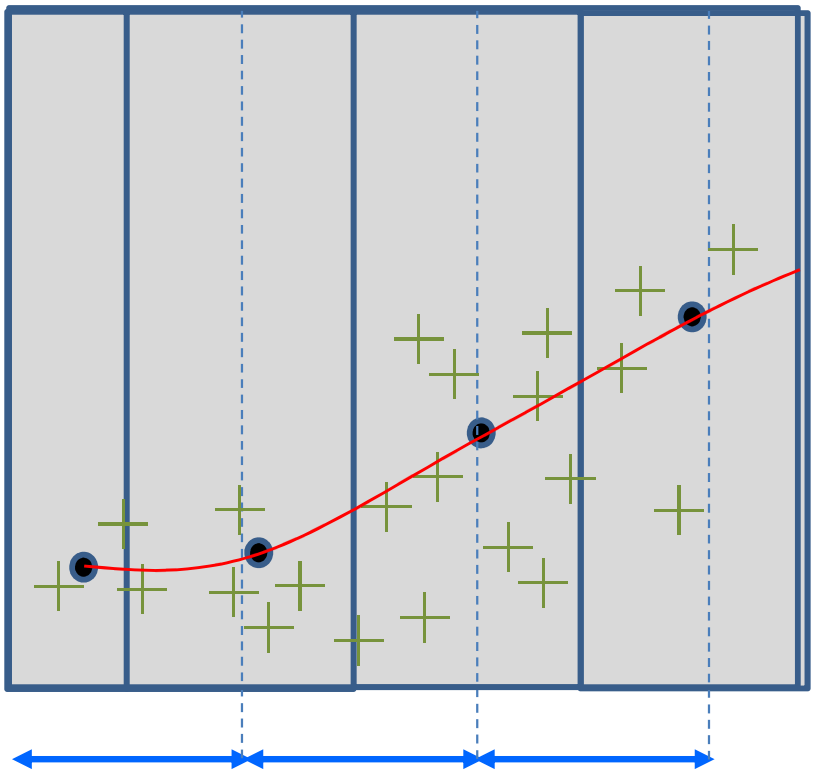
Base map

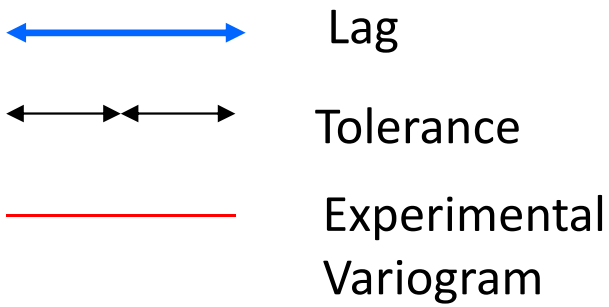


Variogram Cloud

o Experimental variogram for irregular 2-D data set

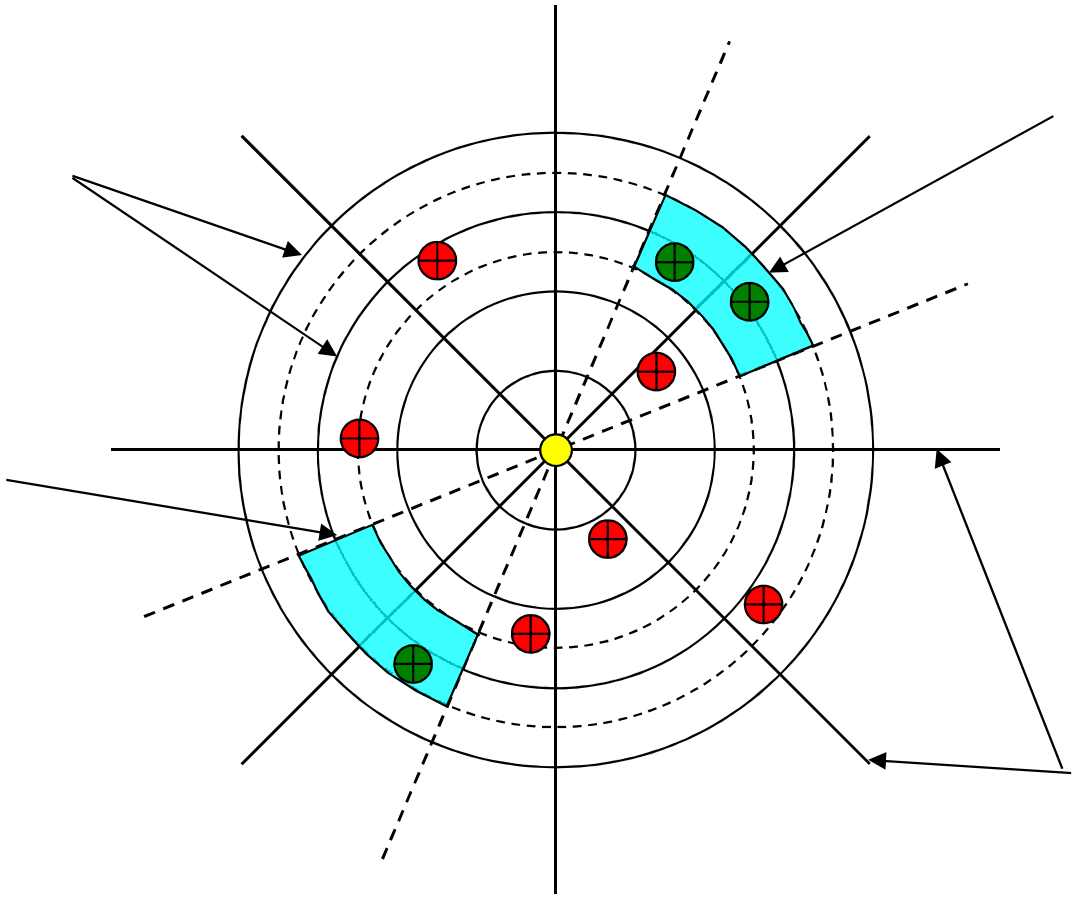
➢ From the variogram cloud to the experimental variogram





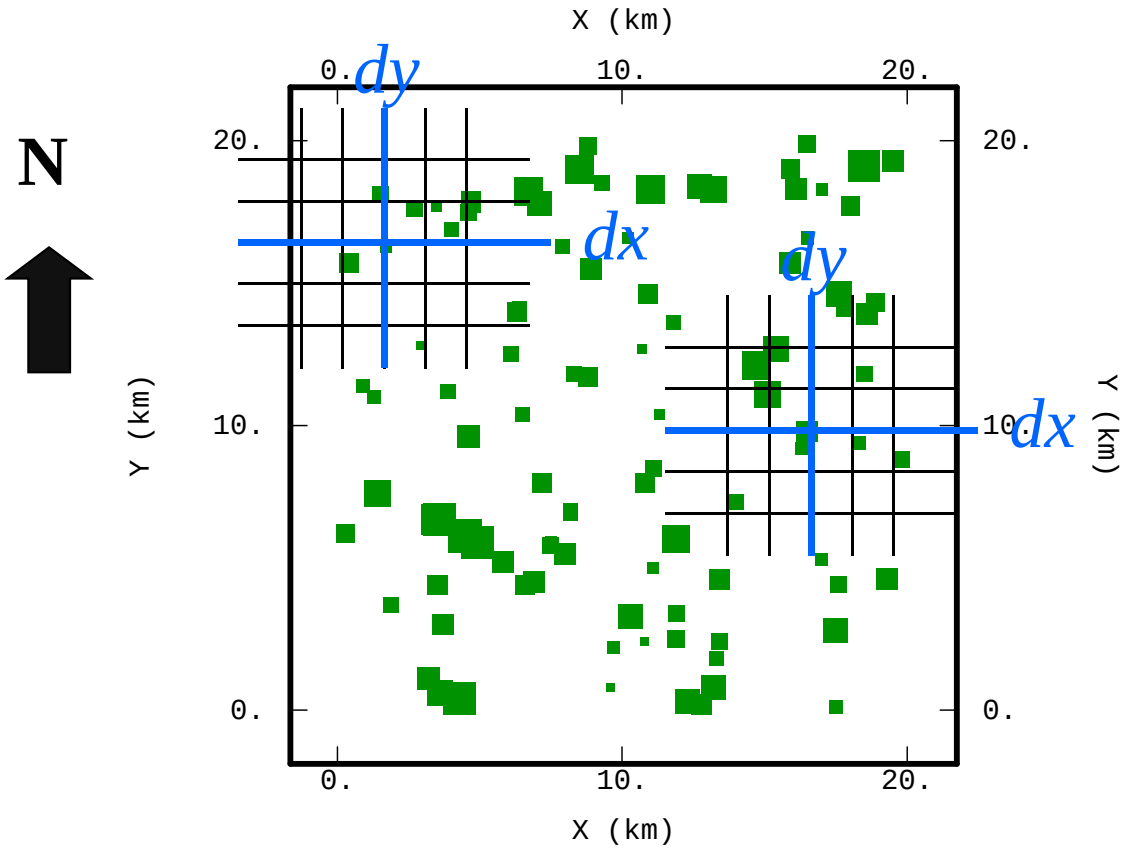
o Experimental directional variograms

- Considering each sample in turn (central point)



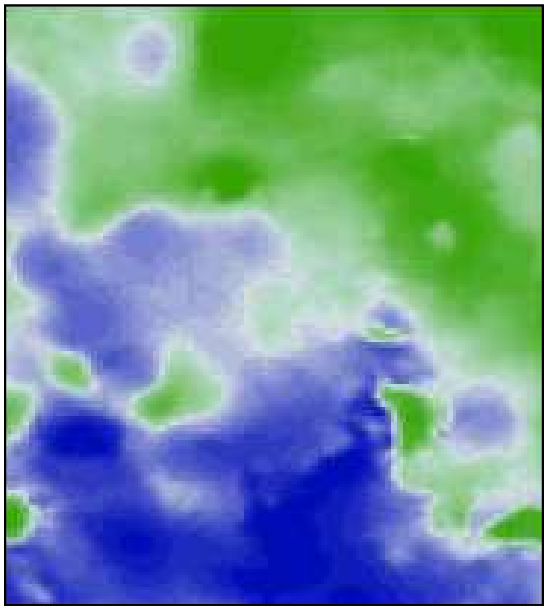
o Variogram Map

- The variogram map shows the variogram as a surface on a regular grid

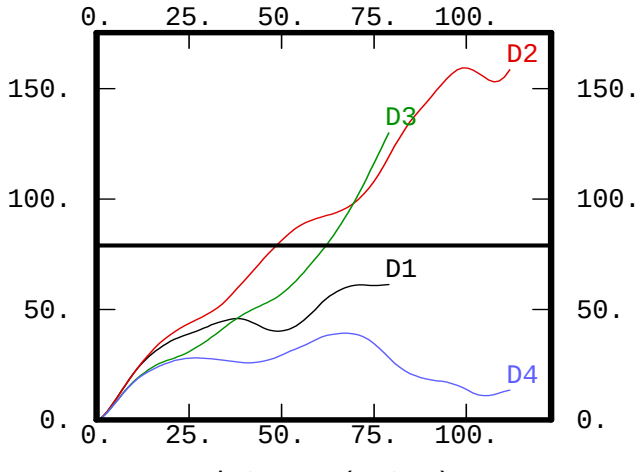


o Variogram map

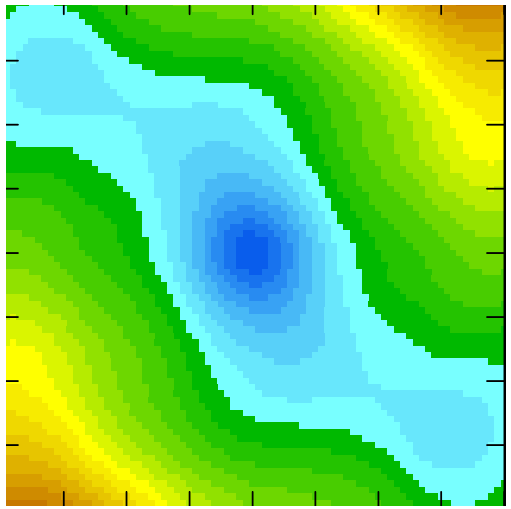
- Illustration using the Vineyard topographic surface



Data



Experimental Variograms calculated in 4 directions



Variogram Map