Simulations

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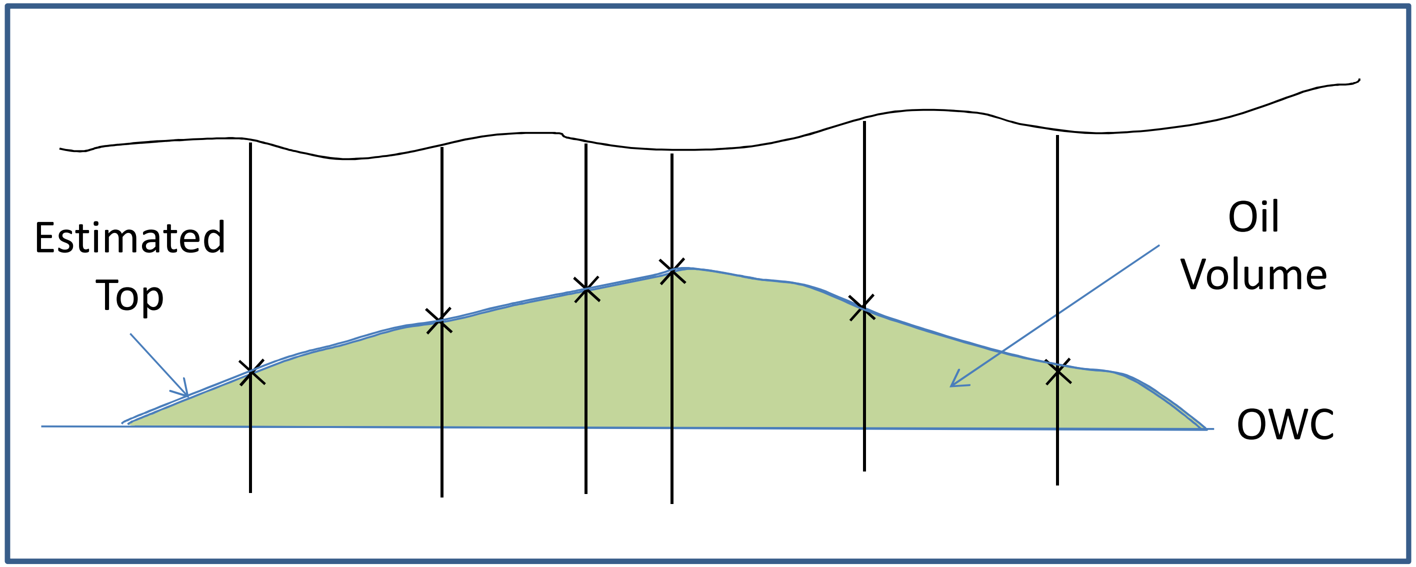
o Why are simulations necessary ?

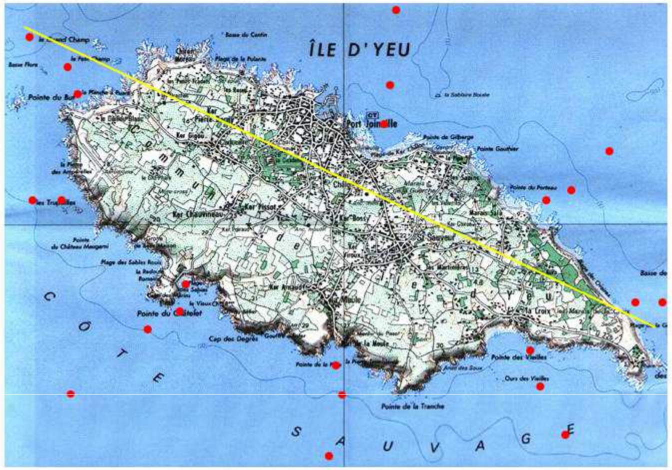
- Estimation (Kriging) produces smooth results
- We need a different method which can:
  - reproduce the variability
  - Give valid (non biased) solution to complex criterion (non linear)
- Example of Volumetrics problem in the Oil industry: get the volume of a reservoir below an impermeable horizon and above the oil-water contact



Yeu Island

Example

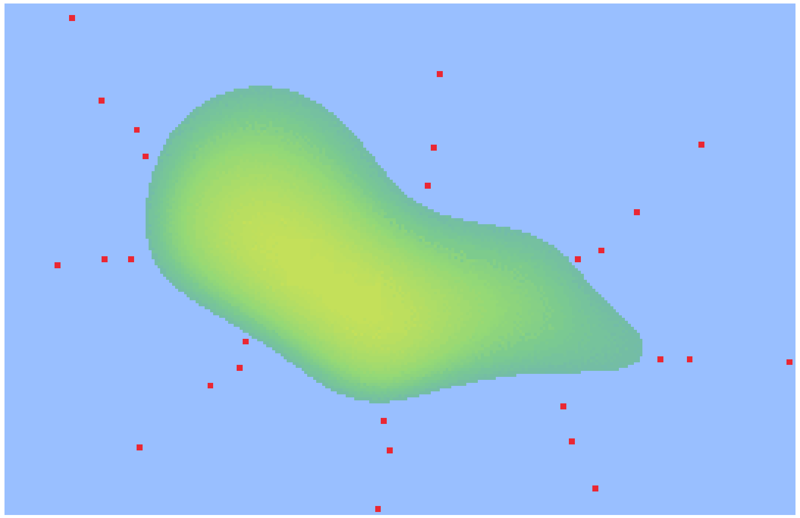
oYeu Island



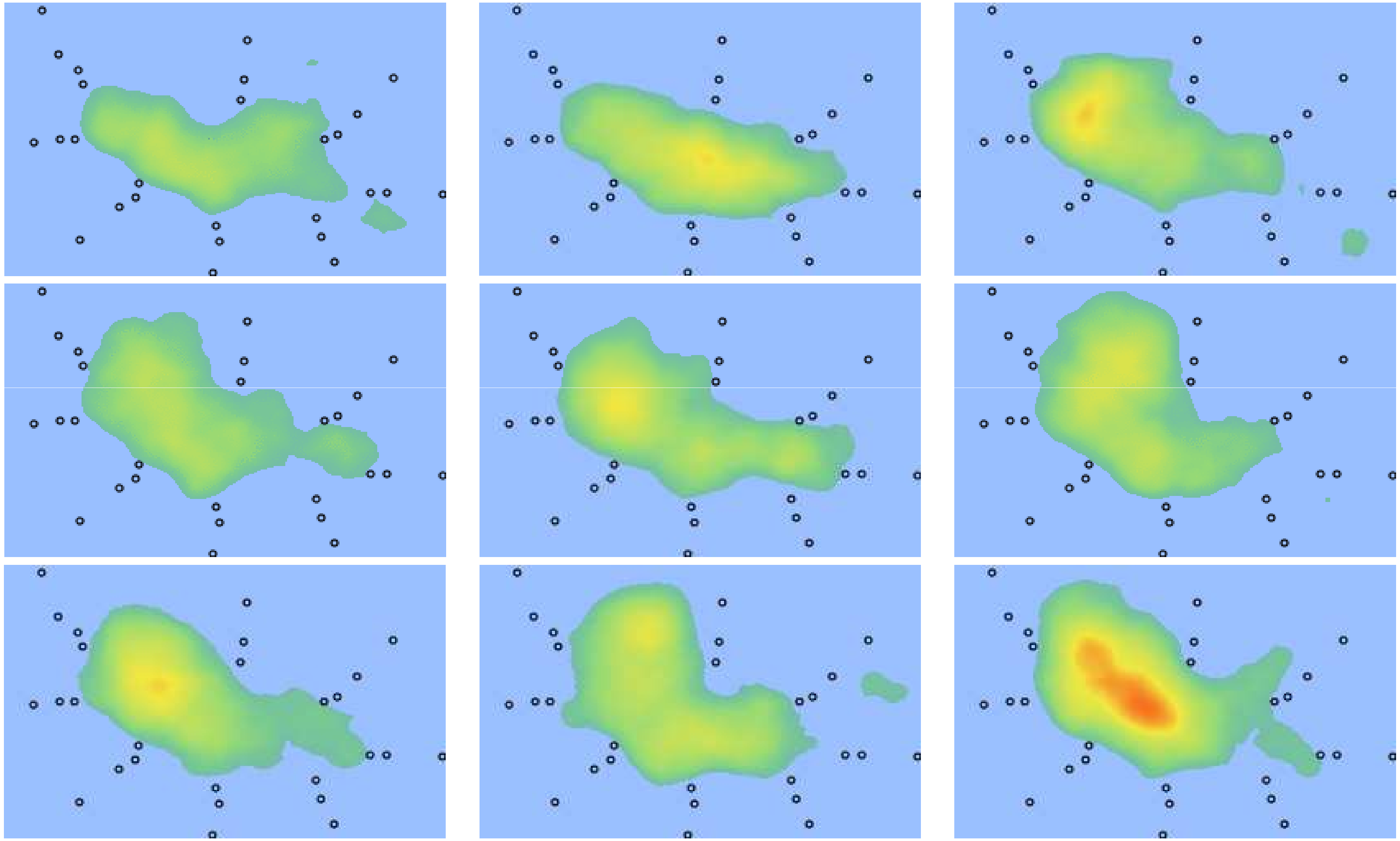
True map
Representation profile

40 samples on 8 bathymetric profiles
No sample ON the island

Samples and Kriged results



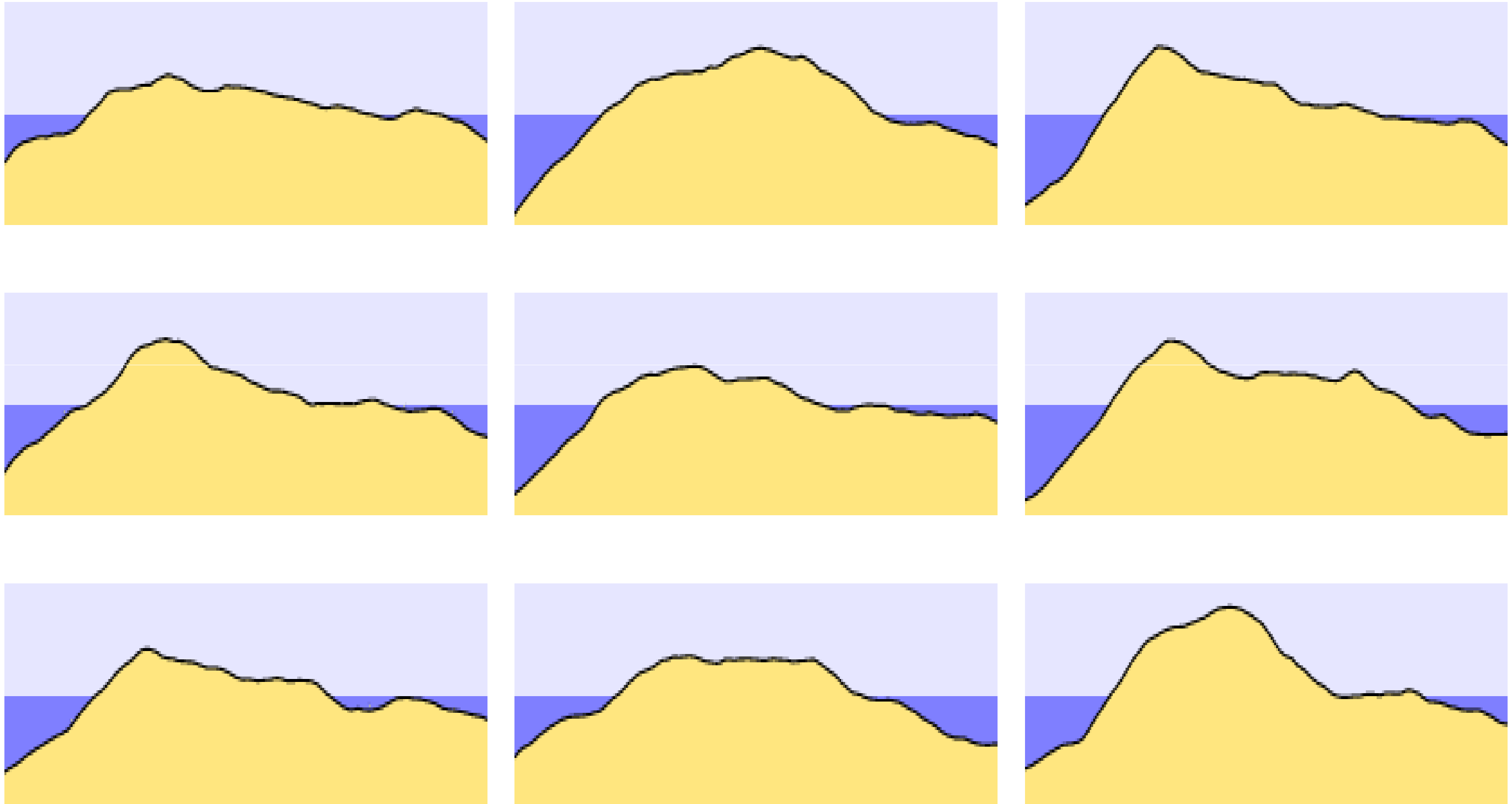
oYeu Island



9 simulations conditioned by the bathymetric profiles

Example

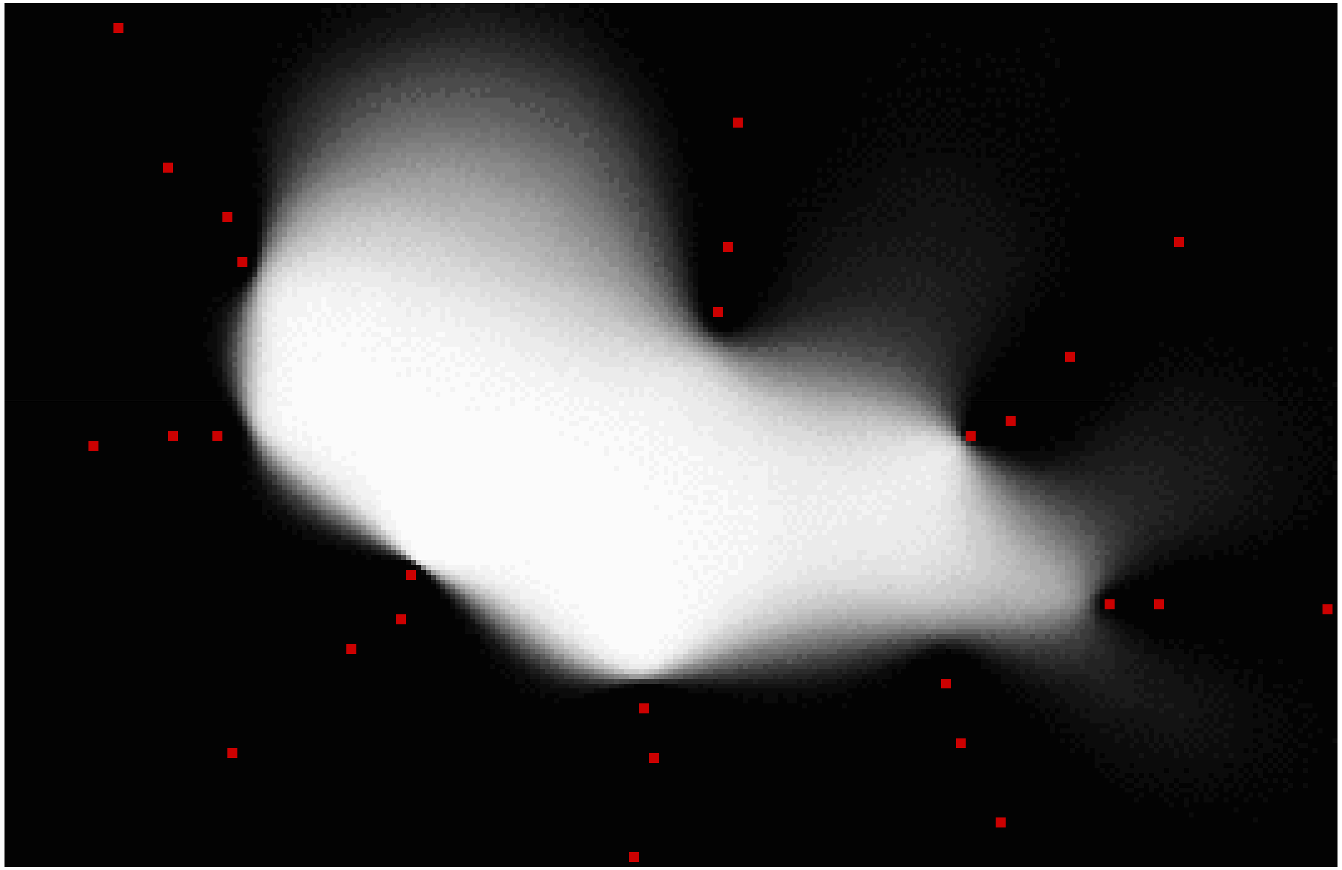
oYeu Island



9 simulated profiles conditioned by the bathymetric profiles

Example

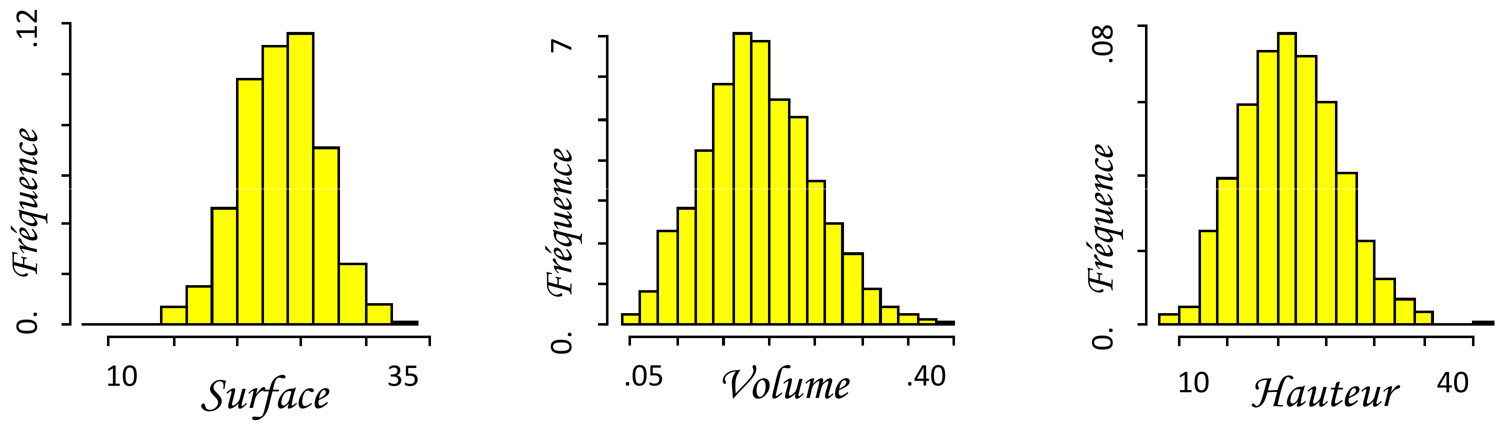
oYeu Island



Probability map to belong to the island

oYeu Island

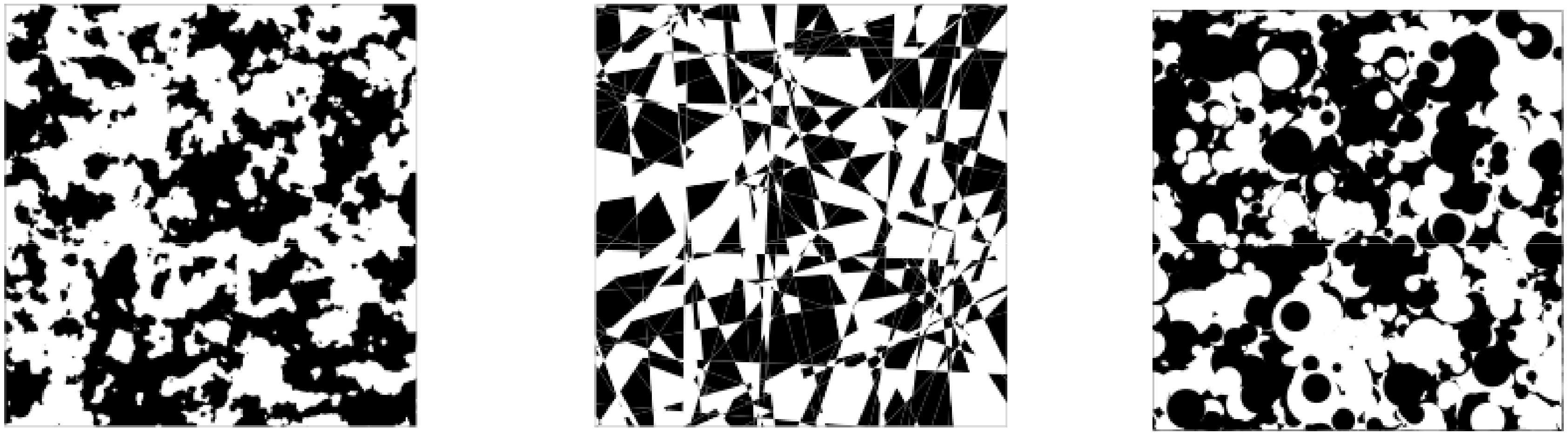
- We can calculate the function of interest per simulation and derive statistics



	Estimation	Simulations	Réalité
Surface (km²)	22.94	23.37	23.32
Volume (km³)	0.169	0.188	
Hauteur (m)	15.93	21.32	27.50

o Spatial Law

- We cannot rely on the first two moments:



Three realizations with same histogram, same covariance, same 3-point statistics

o Spatial Law

- We must know the spatial law which characterizes the variable of interest:

$$P\left(Z(x_1) < z_1, \dots, Z(x_n) < z_n\right) \quad \forall\left(x_1, \dots, x_n\right)$$

- In general, the spatial law is not tractable

Gaussian framework:

- Definition:

$$\{Y(x)\} \text{ gaussian } \Leftrightarrow \left(Y(x_1), \dots, Y(x_n)\right) \text{ gaussian vector}$$

- Simplification in the (multi-) gaussian case:
  - Knowing the first two moments is sufficient to describe the whole spatial law
  - Most of algorithms based on large number of independent replicates tends to normality: *Central Limit Theorem*
  - Stability properties

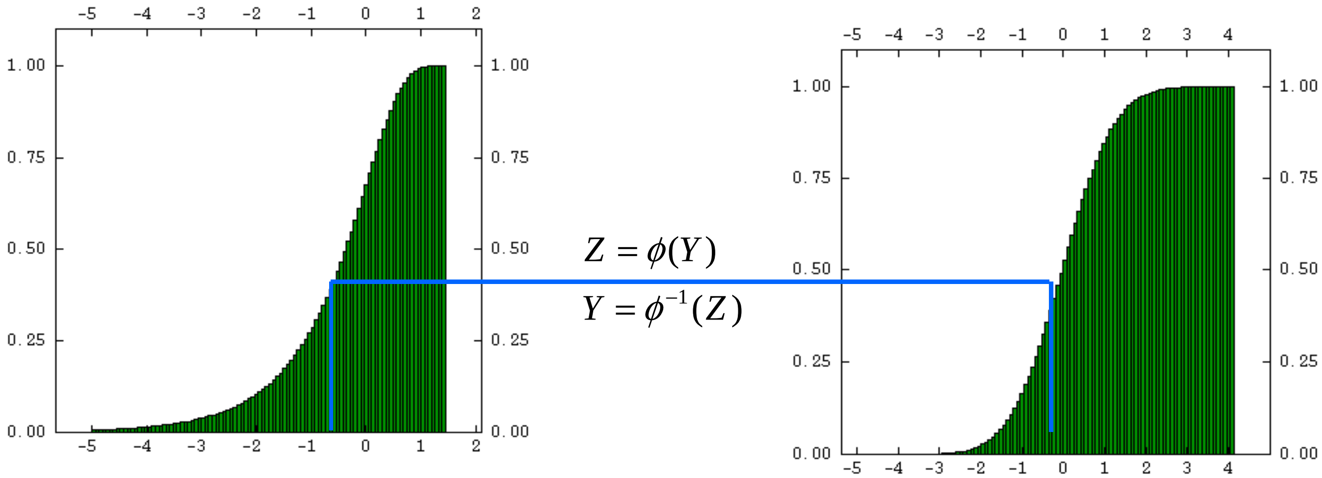
Gaussian Anamorphosis

o Definition

- The variable Z is a Gaussian transformed variable if:

$$Z(x) = \Phi\left[Y(x)\right]$$

ϕ is a monotonous increasing function (called **Gaussian Anamorphosis**)



Raw | Gaussian

- The Gaussian Anamorphosis is fitted using **Hermite** polynomials: it is used to convert simulated results backwards.

o Conditional Law

- Consider the gaussian vector $(Y_0, Y_1, \dots, Y_n)$
- We can write: $Y_0 = Y_{SK} + Y_0 - Y_{SK}$

  where $Y_{SK} = \sum_{\alpha=1}^{n} \lambda_\alpha Y_\alpha$
- Then the following vector is bi-gaussian: $(Y_{SK}, Y_0 - Y_{SK})$
- Orthogonality property of Simple Kriging: $Cov(Y_{SK}, Y_0 - Y_{SK}) = 0$
- Then we can write: $Y_0 = Y_{SK} + \sigma_{SK} G(0,1)$
- Conditional law: $Law(Y_0 \mid Y_1 = y_1, \dots, Y_n = y_n) = G(Y_{SK}, \sigma_{SK})$

o Basic Method

- The conditional law is based on the Simple Kriging of available information
- Hence the simulation basic algorithm:

1 – Draw the first simulated value $Y_S(0)$ according to $G(m,\sigma^2)$

2.1 – Perform Simple Kriging at next target using the previously simulated samples. We obtain Y_S^* and σ_S^2

2.2 – Draw the simulated value according to $G(Y_S^*,\sigma_S^2)$

2.3 – Return to 2.1 until all targets are processed

- Obviously the kriging system grows with the rank of the target.
- This algorithm becomes intractable when the number of targets is large

o Gibbs Sampler

- A similar simulation algorithm:

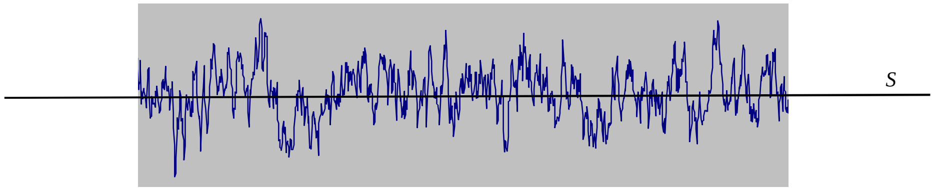
1 – Draw spatial uncorrelated gaussian values at targets according to $G(m,\sigma^2)$
Perform the following iteration several times:
2.1 – Consider one target site at random
2.2 - Perform a simple kriging using all other information. We obtain Y_S^* and σ_S^2
2.3 – Draw the simulated value at target according to $G(Y_S^*,\sigma_S^2)$
2.3 – Iterate 2.1 until all targets have been processed

- This algorithm (also) becomes intractable when the number of targets is large

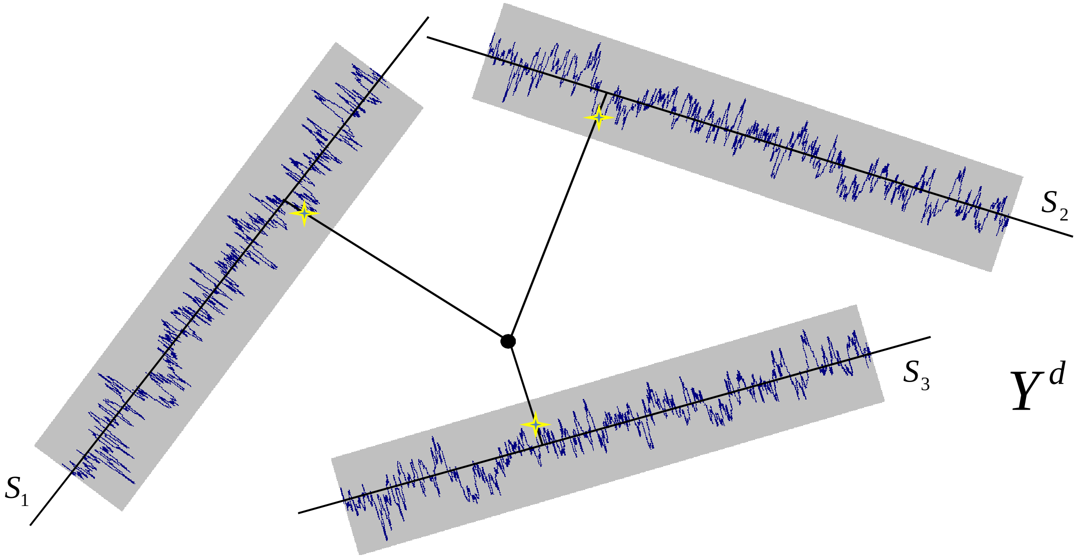
o Turning Bands

Transform the simulation of RF in R^d by several independent simulations in R

- Along one band S, generate the RF $Y^1(s)$ with a given covariance:



- Spread the n bands in R^d



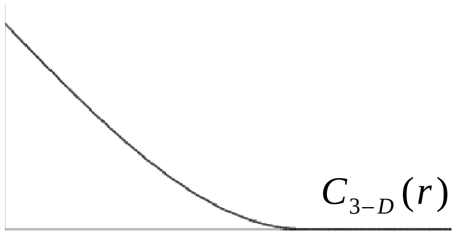
$$Y^d(x) = \frac{1}{\sqrt{n}} \sum_{i=1}^{n} Y^1(< x.S_i >)$$

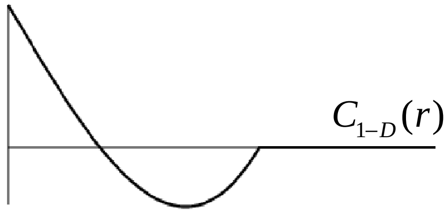
o Spherical model

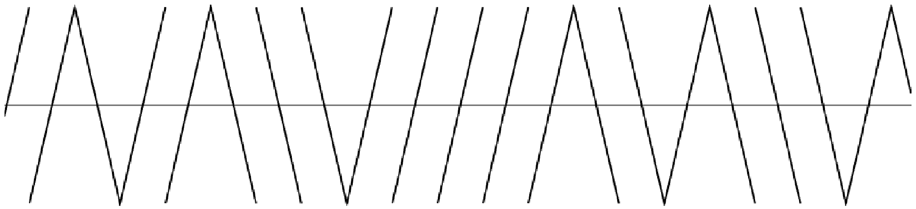
$$C_{3-D}(r) = \left(1 - \frac{3}{2}\frac{|r|}{a} + \frac{1}{2}\frac{|r|^3}{a^3}\right) 1_{|r|\le a}$$

$$\Downarrow$$

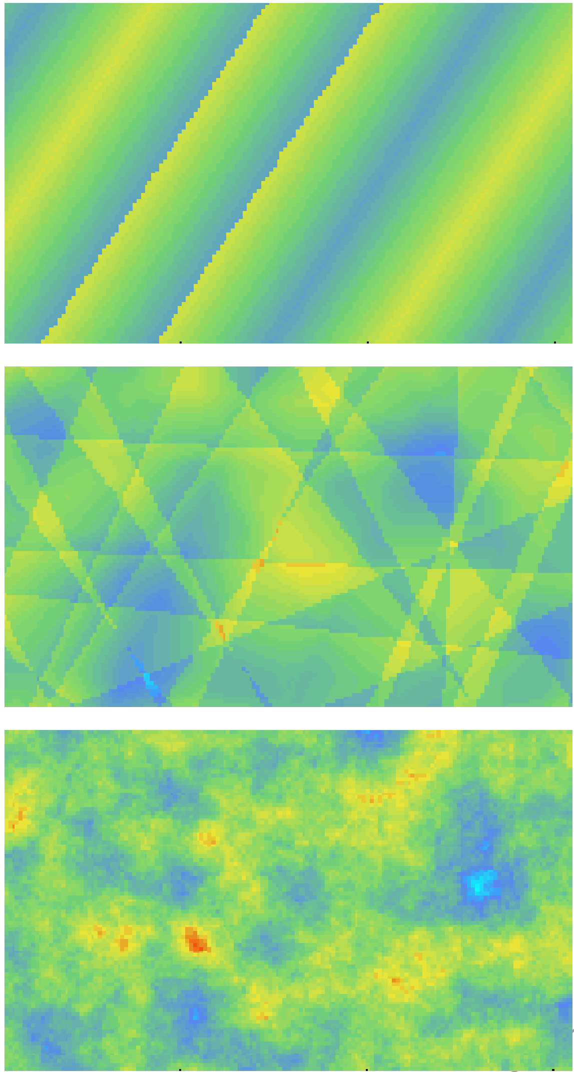
$$C_{1-D}(r) = \left(1 - 3\frac{r}{a} + 2\frac{r^3}{a^3}\right) 1_{r\le a}$$







1-D Simulation



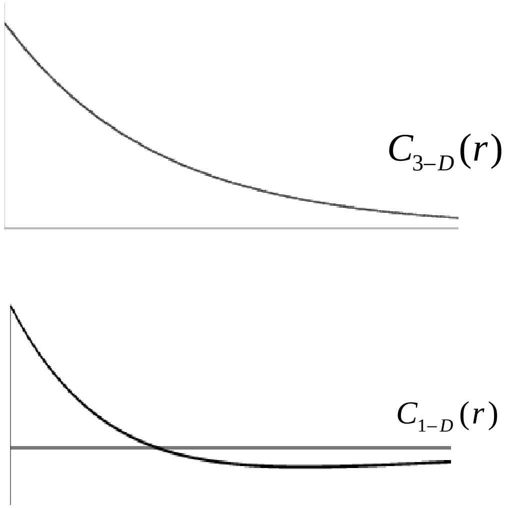
Simulation using turning bands (1, 10, 1000 bands)

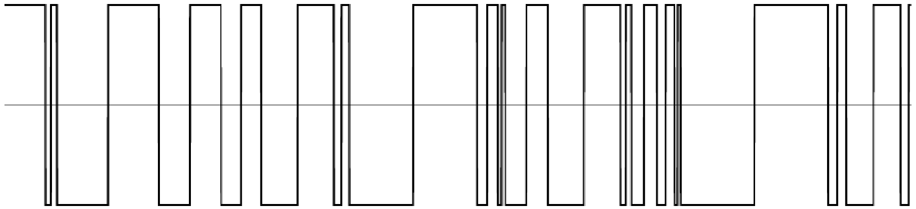
o Exponential model

$$C_3(r) = \exp\left\{-\frac{|r|}{a}\right\}$$

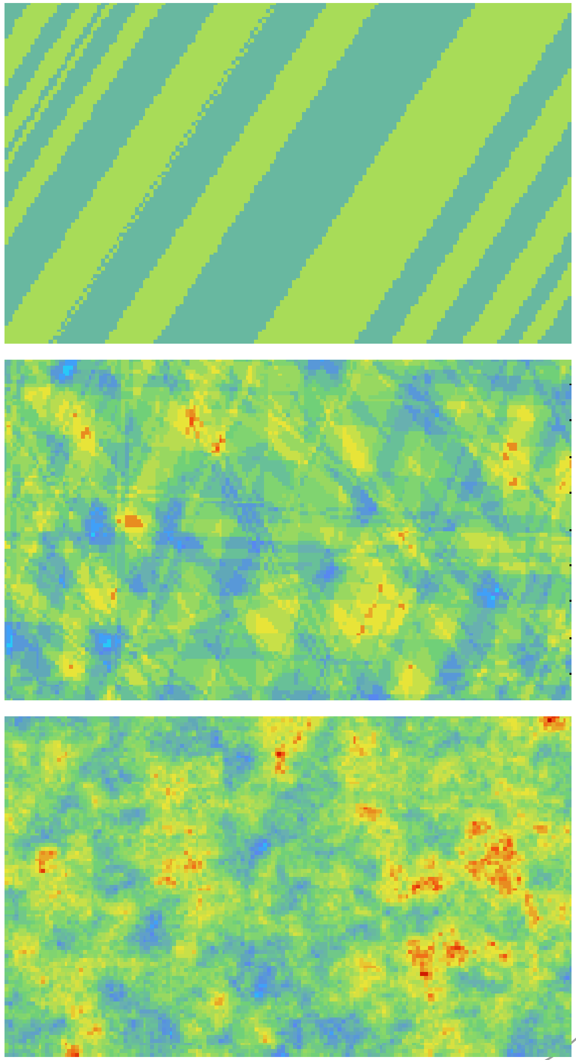
$$\Downarrow$$

$$C_1(r) = \left(1 - \frac{r}{a}\right) \exp\left\{-\frac{r}{a}\right\}$$





1-D Simulation



Simulation using turning bands (1, 10, 1000 bands)