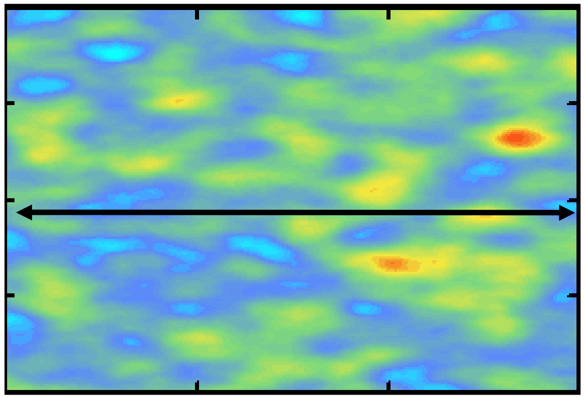
Categorical Simulations

D. Renard

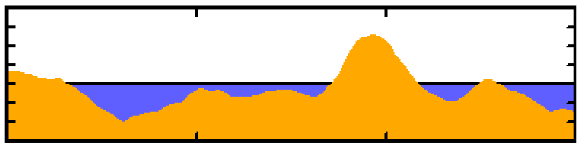
N. Desassis

Truncated Gaussian

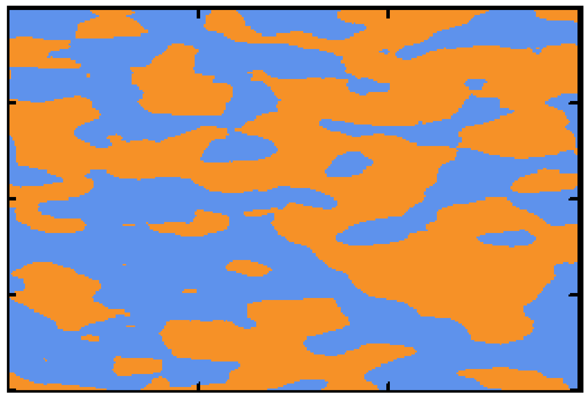
o Principle



Gaussian (stationary) Random Function
with covariance $\rho(h)$



Threshold on the GRF

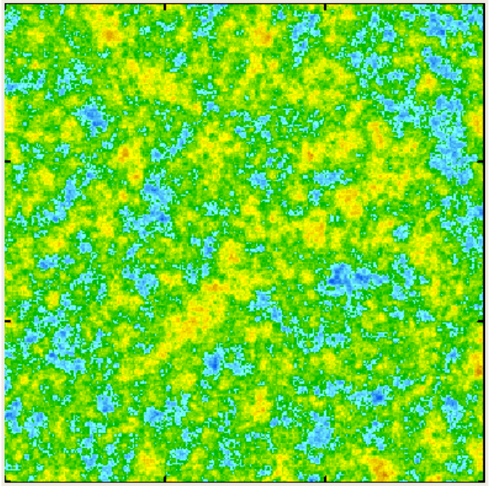


Random Set
Proportion p
Covariance $K(h)$

(Matheron et al. 1987, Galli et al 1994..)

Truncated Gaussian

o Proportions and Thresholds

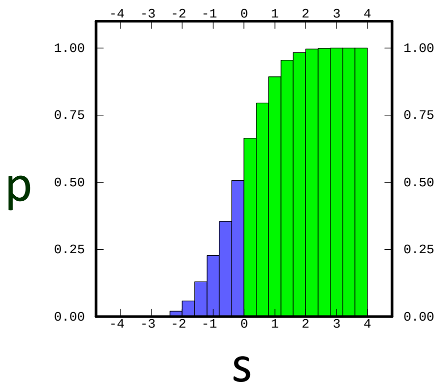


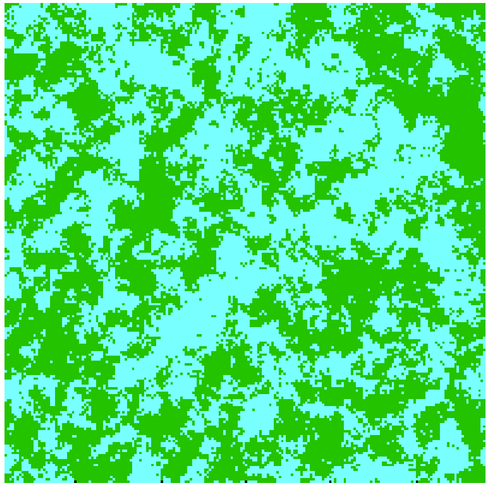
Simulation of a GRF

p = Proportion of blue facies

s = Threshold

Gaussian CDF



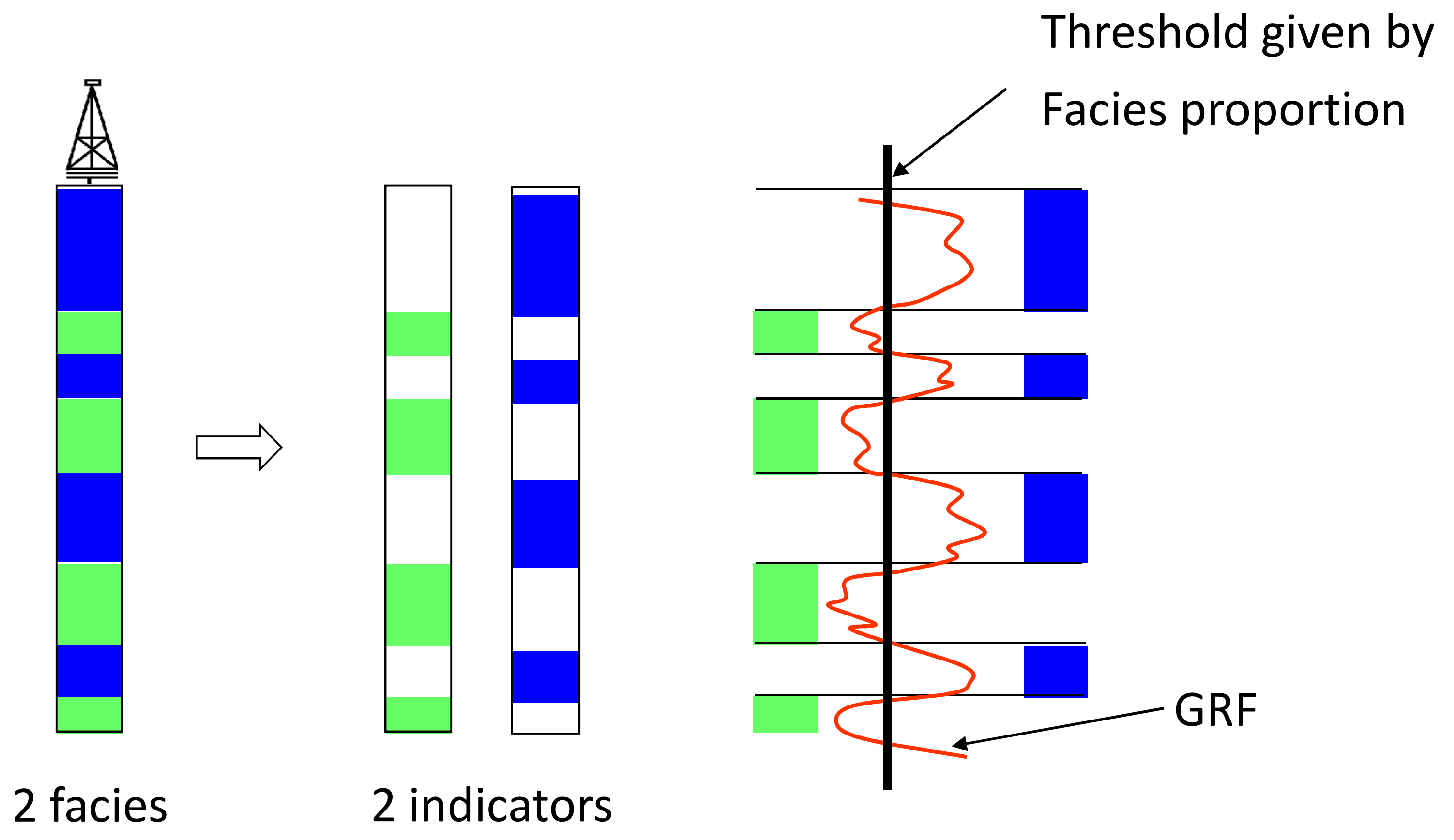


Facies Simulation

Case with 2 facies

Truncated Gaussian

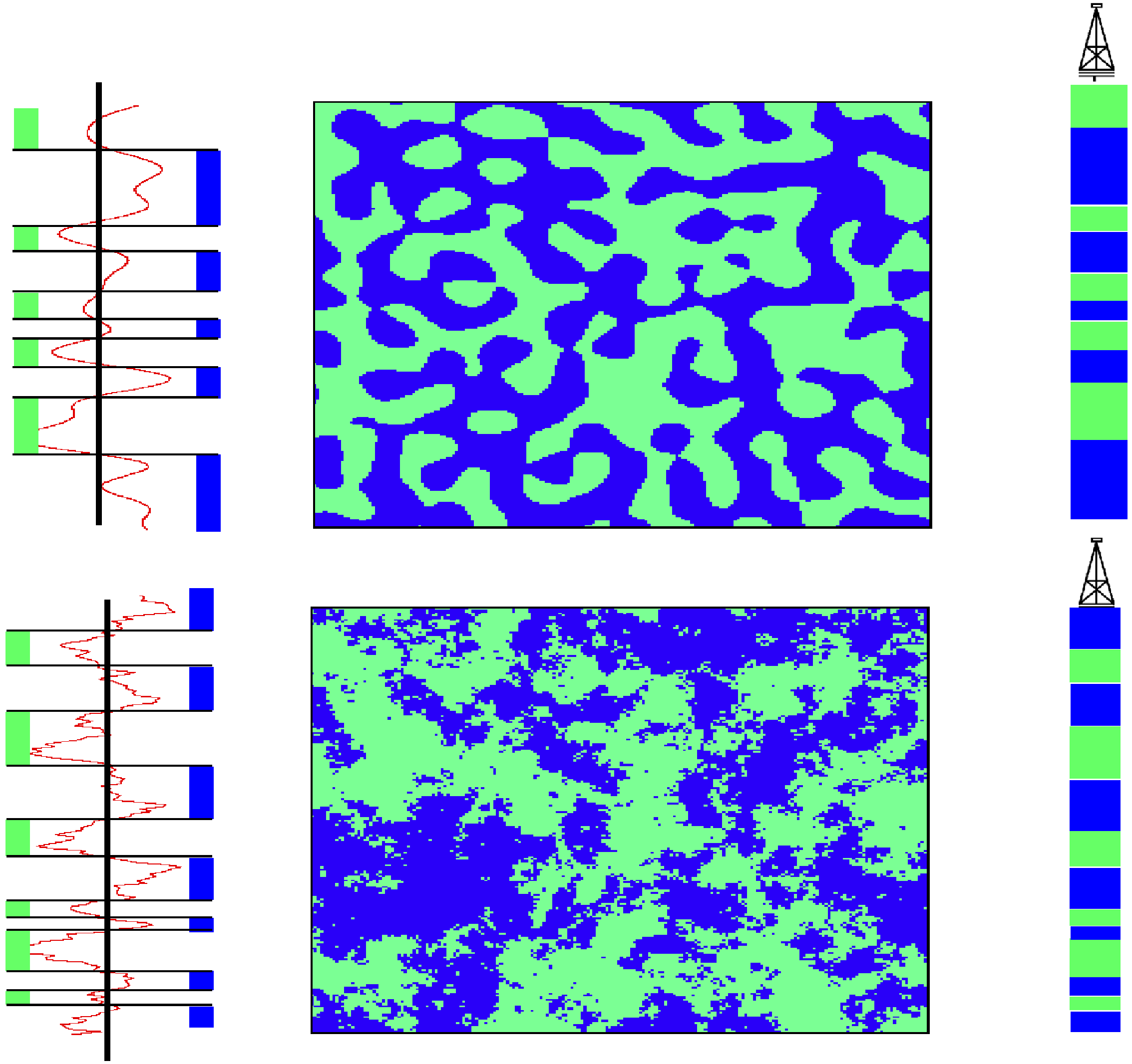
o Threshold and GRF



Truncated Gaussian

o Threshold and GRF

Different models



o Statistics on indicators

- Variance:

$$Var\left(1_A(x)\right) = Var\left(1_{A^C}(x)\right) = P_A(x)\left(1 - P_A(x)\right) \leq 0.25$$

- Non-centered covariance:

$$K_A(h) = E\left(1_A(x)1_A(x+h)\right) = P\left[\left(x \in A\right) et \left(x+h \in A\right)\right]$$

- Non-centered cross-covariance:

$$K_{AA^c}(h) = E\left(1_A(x)1_{A^c}(x+h)\right)$$

- Simple variograms:

$$\gamma_A(h) = \gamma_{A^c}(h) = \frac{1}{2} Var\left[1_A(x) - 1_A(x+h)\right]^2 = p_A - P\left[x \in A \; et \; x+h \in A\right]$$

$$0 \leq \gamma_A(h) \leq 0.5$$

- Cross variograms:

$$\gamma_{AA^c}(h) = -\gamma_A(h) = -\gamma_{A^c}(h)$$

Truncated Gaussian

o Variography

- Link between the (non-centered) covariance of the indicator and the covariance of the underlying GRF

$$K_A(h) = E\left[I_A(x) I_A(x+h)\right]$$

$$K_A(h) = P\left\{\left(Y(x) \leq s\right) \text{ et } \left(Y(x+h) \leq s\right)\right\}$$

$$K_A(h) = \int_{-\infty}^{s}\int_{-\infty}^{s} g_{\rho(h)}(u,v)\,\partial u \partial v \quad \text{with} \quad g_\rho(u,v) = \frac{1}{2\pi\sqrt{1-\rho^2}} e^{-\frac{u^2-2\rho uv+v^2}{2(1-\rho^2)}}$$

- En variogramme

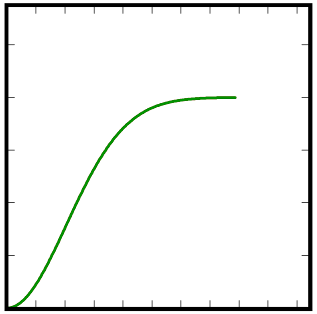
$$\gamma_A(h) = p_A(x) - \int_{-\infty}^{s}\int_{-\infty}^{s} g_{\rho_h}(u,v)\partial u \partial v$$

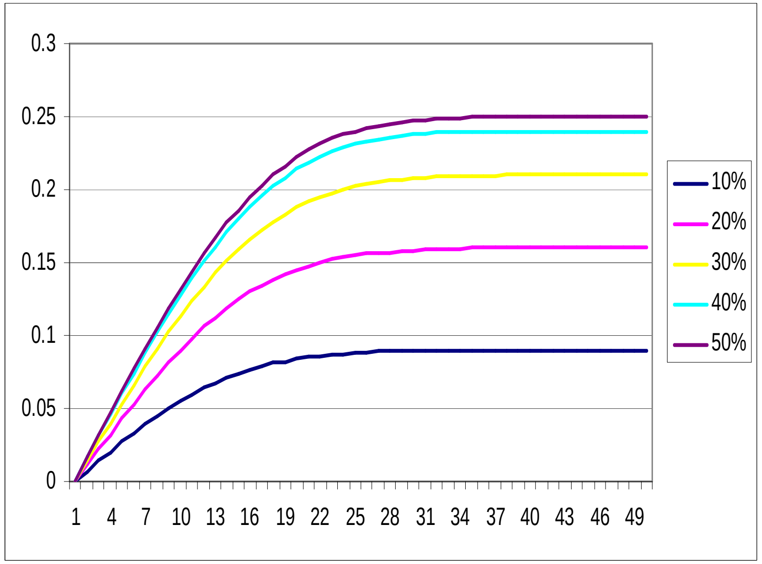
$$\gamma_A(h) = \frac{1}{\pi}\int_0^{\mathrm{Arcsin}\sqrt{\gamma(h)/2}} \exp\left(-\frac{s^2}{2}\left(1+\tan^2 t\right)\right) dt$$

$$\gamma_s(h) \propto \sqrt{\gamma(h)} \qquad \text{for small h}$$

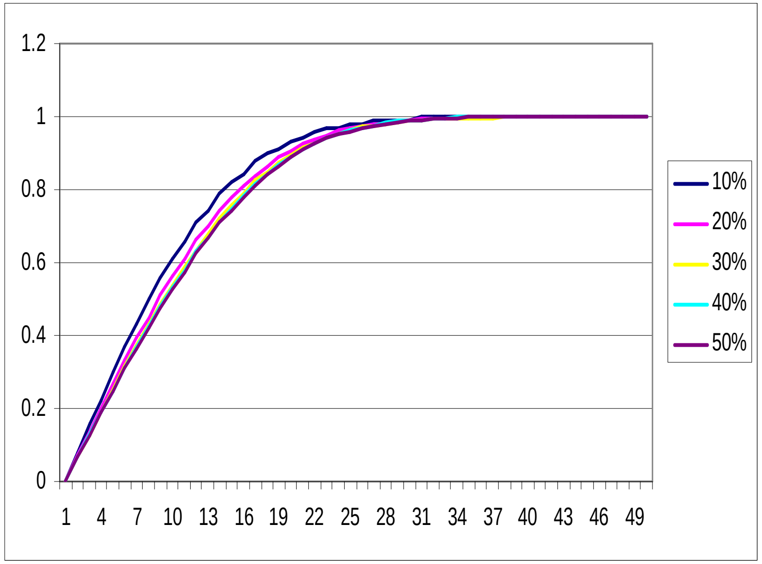
o Variography

Case of an underlying GRF
with gaussian variogram





Indicator variograms



Indicator normalized variograms

o Model Fitting

- Translate facies into indicators (numerical information)

- Calculate the experimental variograms in all directions:
  - N designates the number of facies
  - N*(N+1)/2 simple and cross variograms

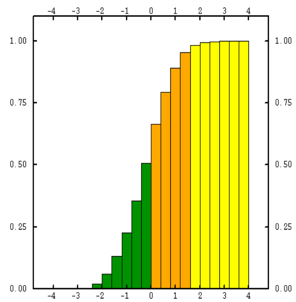
- Guess the model of the underlying GRF

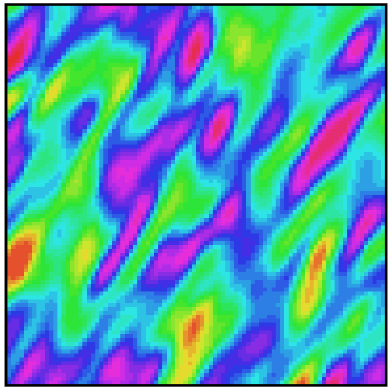
Truncated Gaussian

o One GRF – Three facies

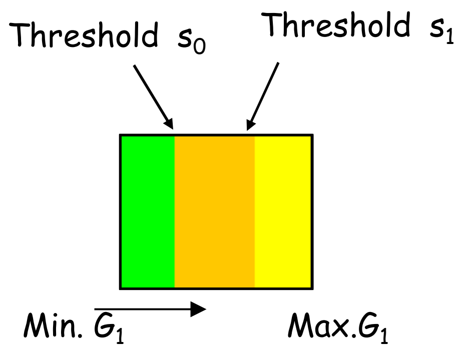
$p_2+p_1 = G^{-1}(s_1)$

$p_1 = G^{-1}(s_0)$

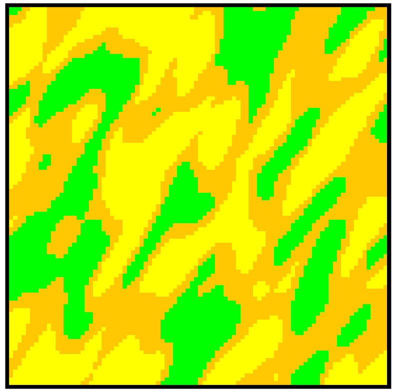




Gaussian RF



Lithotype rule

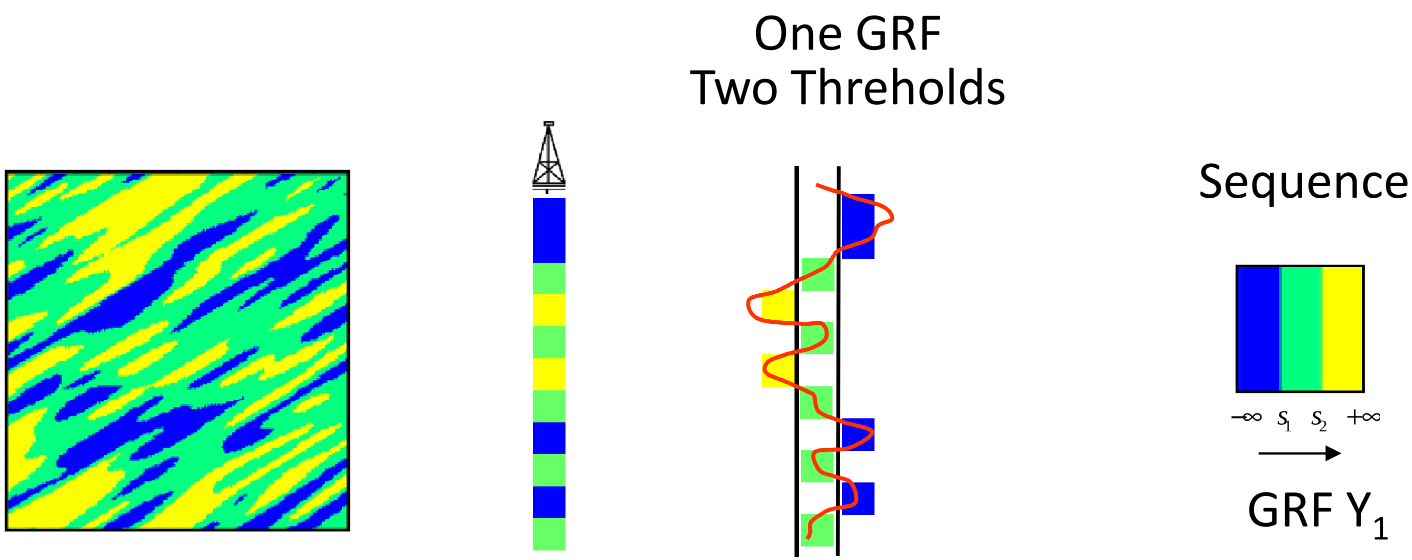


Facies Simulation

Truncated Gaussian

o One GRF – Three facies

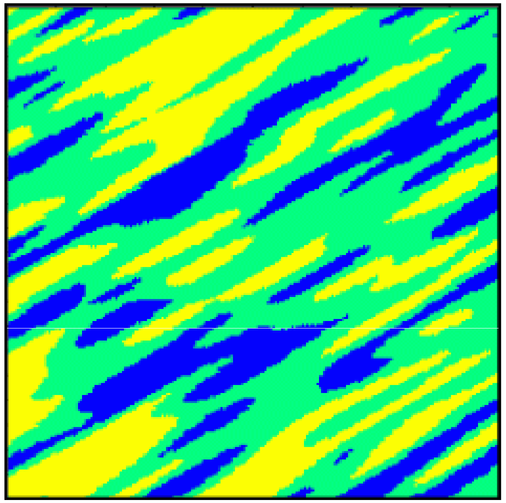
- Facies are ordered. There is **a border effect** when
  - Going from blue to yellow, we must transit in green



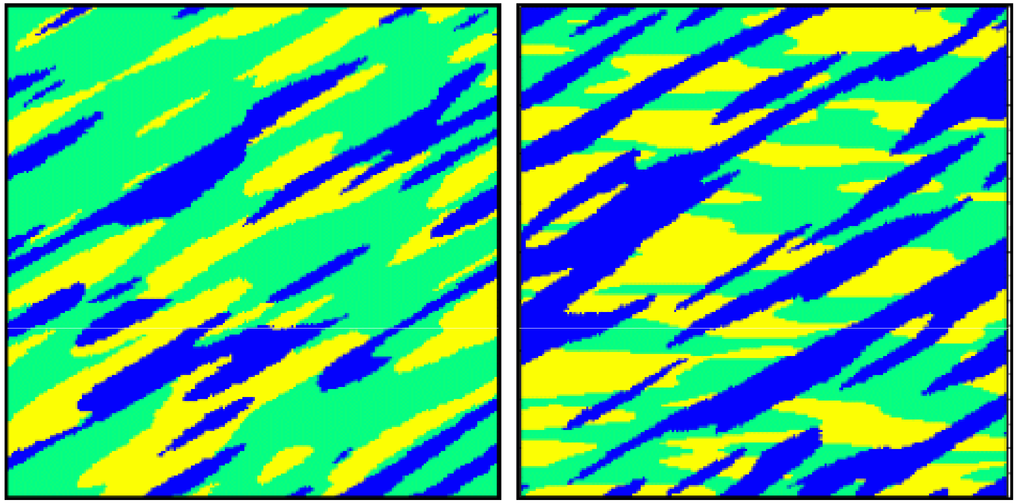
From Mono to PluriGaussian

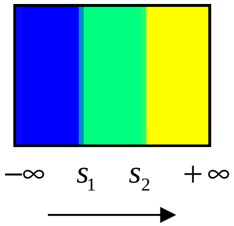
o Need for more ?

Ordered

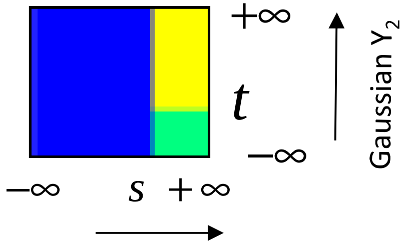


Erosion



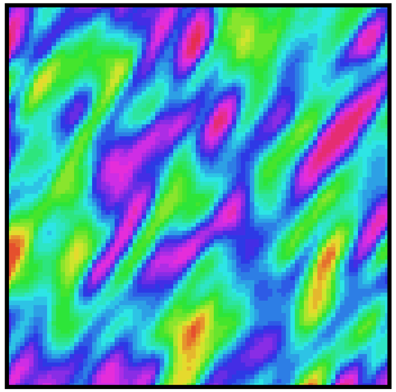


Gaussian Y_1

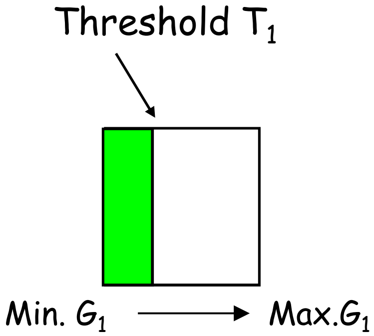


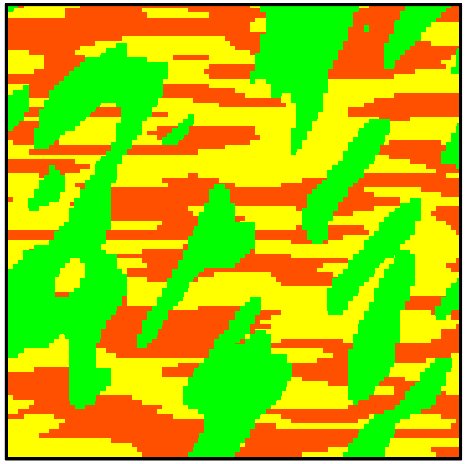
Gaussian Y_1

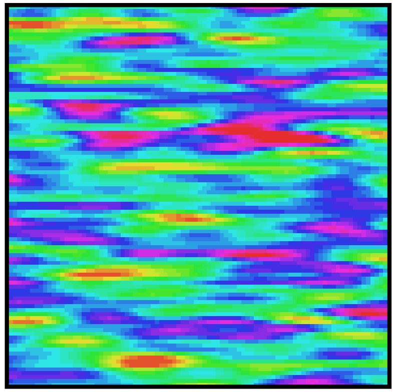
o Three facies – Two GRF



Gaussian (G_1)

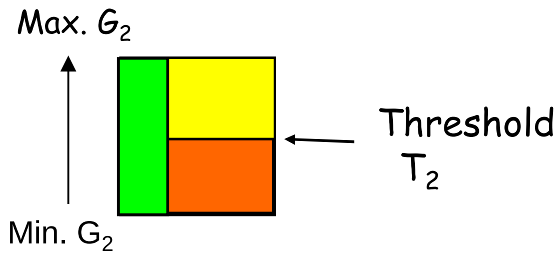




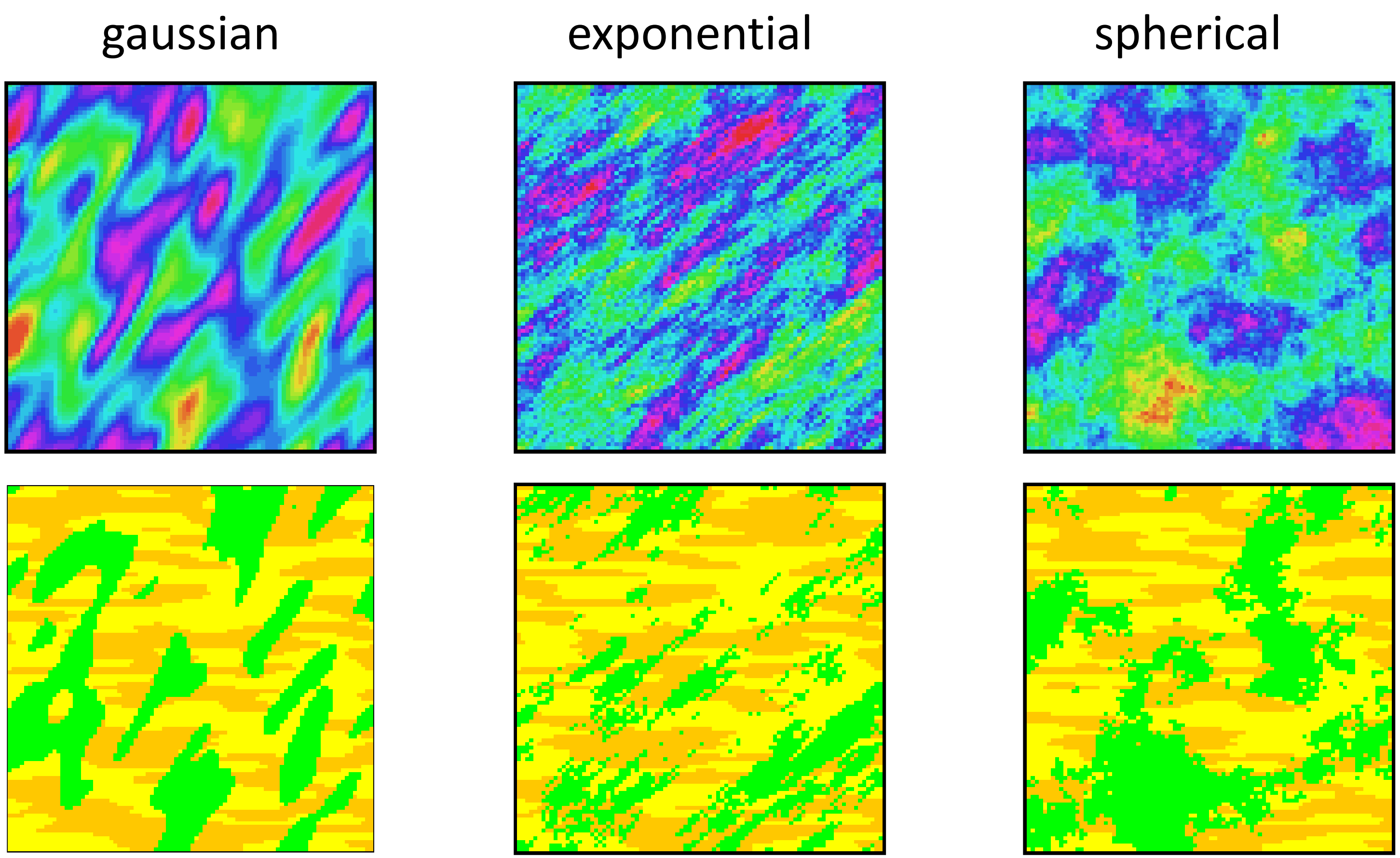


Gaussian (G_2)

Facies Simulation

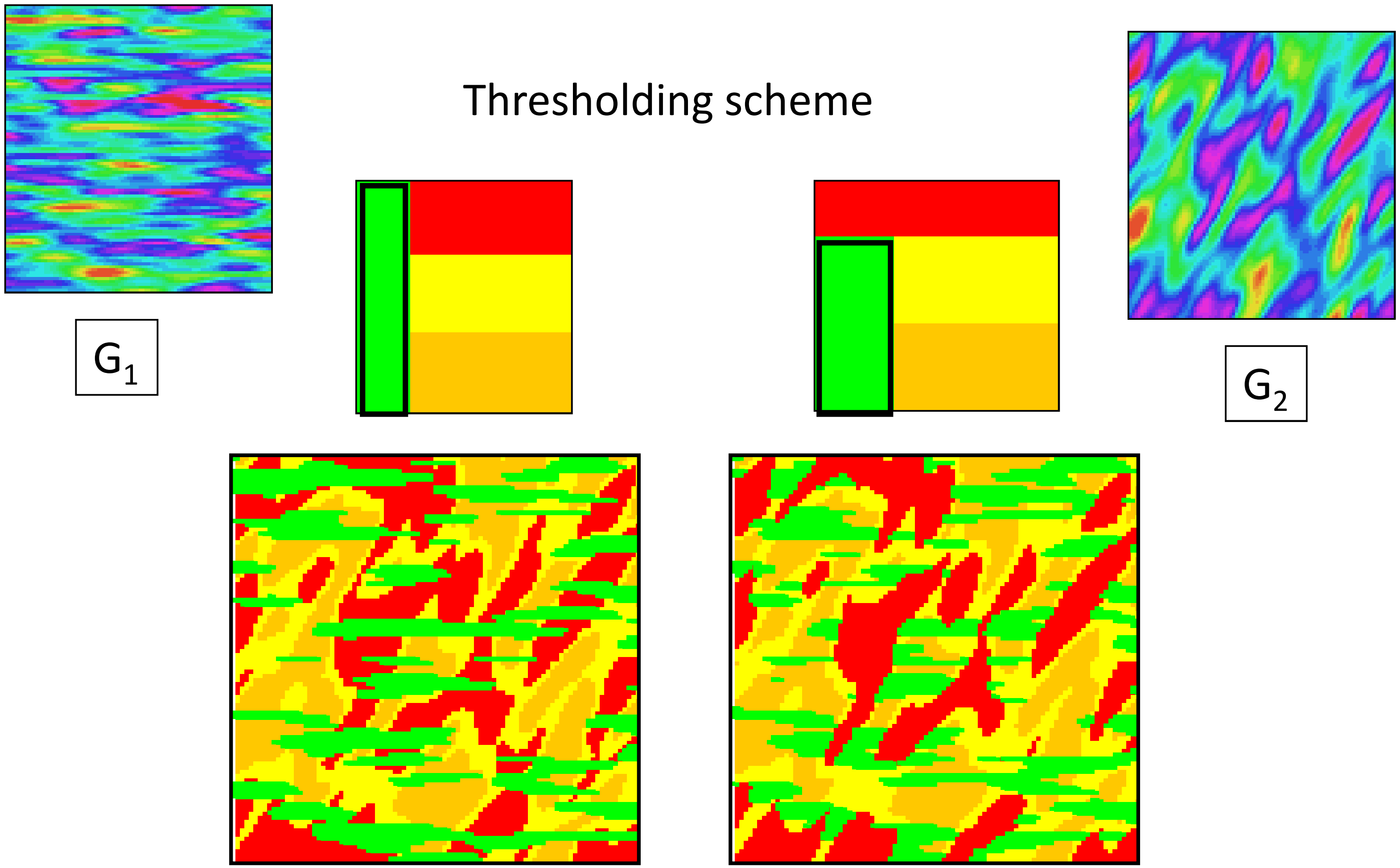


o Different variogram types

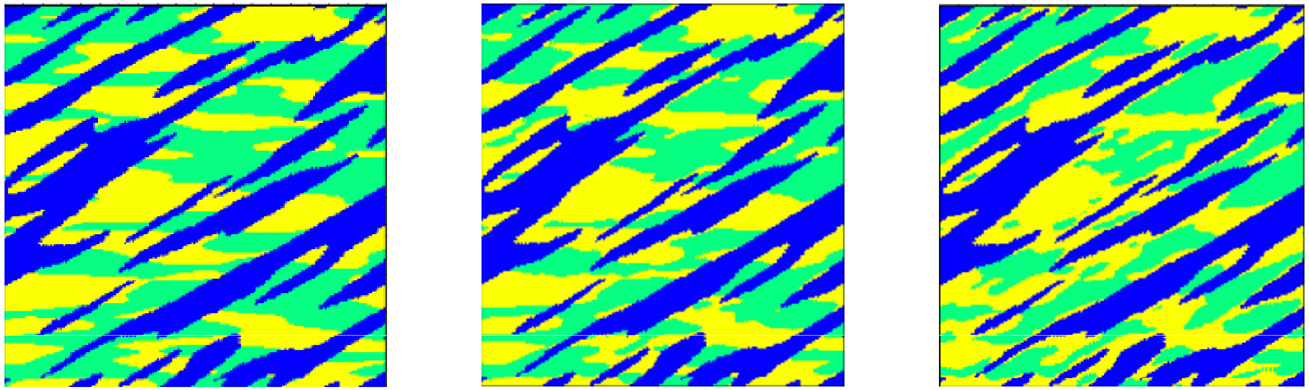


Facies Simulations (different G_1 - same G_2)

o Influence of the Threshold



o Correlated underlying GRF



$\rho = 0.$ $\rho = 0.4$ $\rho = 0.8$

The underlying gaussian RF are intrinsically correlated:

$$\begin{cases} Y_1(x) = Z_1(x) \\ Y_2(x) = \rho Z_1(x) + \sqrt{(1-\rho^2)}\, Z_2(x) \end{cases}$$

Z_1 and Z_2 not correlated

Truncated Gaussian

o Conditioning

- Data are given in facies and must be translated in gaussian values first: Gibbs sampler

$$Y(x_i) = Y^*(x_i) + \sigma R(x_i)$$

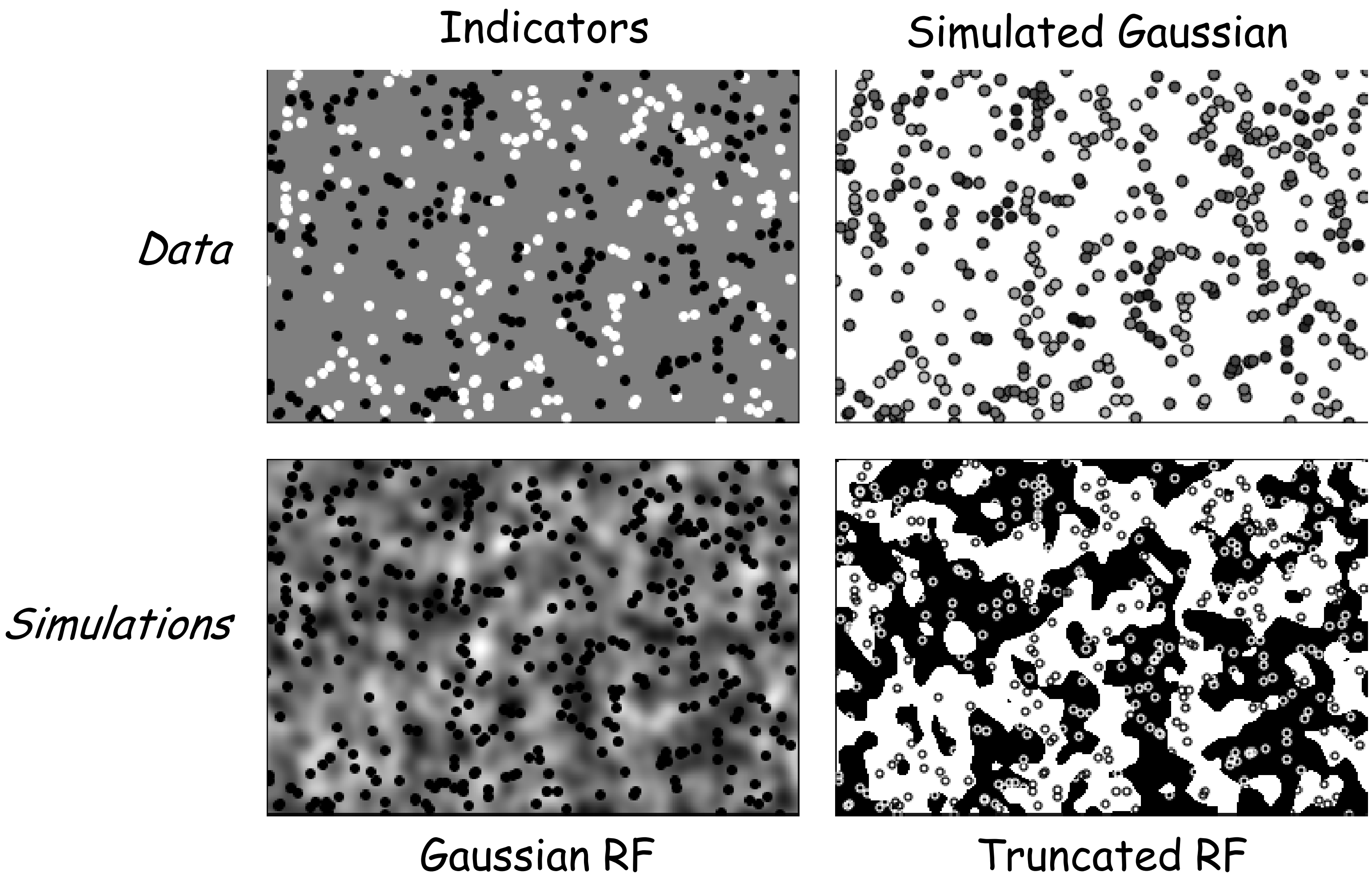
As sample x_i belongs to a given facies, then $Y(x_i) \in \left[s_i^1, s_i^2 \right]$

We must simply draw the gaussian residual such that:

$$\frac{s_i^1 - Y^*(x_i)}{\sigma} < R(x_i) \leq \frac{s_i^2 - Y^*(x_i)}{\sigma}$$

Truncated Gaussian

o Conditioning



o Conditioning