

RGEOSTATS TUTORIAL

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ORGANIZATION

- 4 main tutorials
 - Variography
 - Estimation (kriging)
 - Multivariate Geostatistics
 - Simulations (part I-II-III)

ORGANIZATION

- 4 main tutorials
 - Variography
 - Estimation (kriging)
 - Multivariate Geostatistics
 - Simulations (part I-II-III)
- For each tutorial
 - An ascii (text) file is available
 - Preamble
 - Several independent parts
 - Copy-Paste each line
 - You should **run the preamble** before each part!
 - Some parts are indicated as “see also”
 - You can also play by yourself

SCOTLAND JANUARY TEMPERATURES

- The data set contains temperatures of January measured on 151 stations and averaged over twenty years (1961-1980)¹
- The data set will be used for the first three parts

¹Hudson, G. and Wackernagel, H. (1994) Mapping temperature using kriging with external drift: theory and example from Scotland. *International Journal of Climatology* 14, 77-91.

1) VARIOGRAPHY

INTRODUCTION

- 1 Preamble: load and prepare the data
- 2 Exploratory data analysis
- 3 Compute and plot an omnidirectional experimental variogram
- 4 Fit a model
- 5 Several directions
- 6 Variogram map
- 7 Variogram for data on a grid

1) VARIOGRAPHY

1. PREAMBLE

```
# Load the data
# Change the name (to have a shorter name)
# Make the variable 4 (Elevation) inactive

data(Exdemo_Scotland_Temperatures)
dat=Exdemo_Scotland_Temperatures
dat=db.locate(dat,4)
```

1) VARIOGRAPHY

1. PREAMBLE

```
# Load the data
# Change the name (to have a shorter name)
# Make the variable 4 (Elevation) inactive

data(Exdemo_Scotland_Temperatures)
dat=Exdemo_Scotland_Temperatures
dat=db.locate(dat,4)

#Last line equivalent to
dat=db.locate(dat,4,"NA")
dat=db.locate(dat,"Elevation","NA")
#...
```


1) VARIOGRAPHY

2. EXPLORATORY DATA ANALYSIS

```
?db.plot
```

```
dat
```

```
dat []
```

```
hist(dat[,5])
```

```
db.stat(dat,fun="maxi",name=5)
```

```
plot(dat)
```

```
plot(dat,scalefactor=1)
```

```
plot(dat,scale=1)
```

```
plot(dat,scale=1,name.post=5)
```

1) VARIOGRAPHY

3. EXPERIMENTAL VARIOGRAM

```
#For documentation on vario-class  
class?vario
```

1) VARIOGRAPHY

3. EXPERIMENTAL VARIOGRAM

```
#For documentation on vario-class  
class?vario  
  
vario=vario.calc(dat,lag=10,nlag=40)
```

1) VARIOGRAPHY

3. EXPERIMENTAL VARIOGRAM

```
#For documentation on vario-class  
class?vario  
  
vario=vario.calc(dat,lag=10,nlag=40)  
  
?vario.plot
```

1) VARIOGRAPHY

3. EXPERIMENTAL VARIOGRAM

```
#For documentation on vario-class
class?vario

vario=vario.calc(dat,lag=10,nlag=40)

?vario.plot

plot(vario)
plot(vario,npairdw=TRUE)
plot(vario,npairdw=TRUE,npairpt=TRUE)
```

1) VARIOGRAPHY

4. FITTING

```
#For documentation on model-class  
class?model
```

1) VARIOGRAPHY

4. FITTING

```
#For documentation on model-class  
class?model  
  
model=model.auto(vario)
```

1) VARIOGRAPHY

4. FITTING

```
#For documentation on model-class  
class?model
```

```
model=model.auto(vario)
```

```
melem.name()
```

```
melem.name(c(1,2,3))
```


1) VARIOGRAPHY

4. FITTING

```
#For documentation on model-class  
class?model
```

```
model=model.auto(vario)
```

```
melem.name()
```

```
melem.name(c(1,2,3))
```

```
model.auto(vario,struct=melem.name(c(1,2,3)))
```

```
model.auto(vario,struct=melem.name(c(1,3,2)))
```

1) VARIOGRAPHY

4. FITTING

```
#For documentation on model-class  
class?model
```

```
model=model.auto(vario)
```

```
melem.name()
```

```
melem.name(c(1,2,3))
```

```
model.auto(vario,struct=melem.name(c(1,2,3)))
```

```
model.auto(vario,struct=melem.name(c(1,3,2)))
```

```
model.auto(vario,struct=melem.name(c(1,3,2)),  
           maxiter=0)
```

1) VARIOGRAPHY

4. FITTING (WITH CONSTRAINTS)

```
model.auto(vario,struct=melem.name(c(1,2,3)),  
           flag.noreduce=T)
```

1) VARIOGRAPHY

4. FITTING (WITH CONSTRAINTS)

```
model.auto(vario,struct=melem.name(c(1,2,3)),  
           flag.noreduce=T)
```

```
model.auto(vario,struct=melem.name(c(1,2,3)),  
           flag.noreduce=T,verbose=T)
```

1) VARIOGRAPHY

4. FITTING (WITH CONSTRAINTS)

```
model.auto(vario,struct=melem.name(c(1,2,3)),  
           flag.noreduce=T)
```

```
model.auto(vario,struct=melem.name(c(1,2,3)),  
           flag.noreduce=T,verbose=T)
```

```
model.auto(vario,struct=melem.name(c(1,2,3)),  
           flag.noreduce=T,  
           lower=c(0.02,NA,NA,NA,NA),  
           upper=c(0.02,NA,NA,NA,400))
```

1) VARIOGRAPHY

5. SEVERAL DIRECTIONS

```
vario_4dir=vario.calc(dat,lag=15,nlag=15,  
                      dir=c(0,45,90,135))
```

```
vario_4dir
```

```
m4dir=model.auto(vario_4dir,  
                  struct=melem.name(c(1,3,2)))
```

1) VARIOGRAPHY

5. SEVERAL DIRECTIONS

```
#Display results
```

```
plot(vario_4dir,npairdw=TRUE)
```

```
plot(m4dir,add=T,vario=vario_4dir)
```

1) VARIOGRAPHY

6. VARIOGRAM MAP

```
vm=vmap.calc(dat,nx=10,dx=15)  
plot(vm)
```


1) VARIOGRAPHY

6. VARIOGRAM MAP

```
vm=vmap.calc(dat,nx=10,dx=15)  
plot(vm)
```

```
model=vmap.auto(vm,struct=melem.name(c(1,3,2)),  
                flag.noreduce=T,  
                lower=c(0.02,NA,NA,NA,NA),  
                upper=c(0.02,NA,NA,NA,NA))
```

1) VARIOGRAPHY

6. VARIOGRAM MAP

```
vario_4dir=vario.calc(dat,lag=15,nlag=15,  
                      dir=c(0,45,90,135))
```

```
plot(vario_4dir)
```

```
plot(model,vario=vario_4dir,add=T)
```

1) VARIOGRAPHY

7. VARIOGRAM FOR GRID

```
data(Exdemo_Scotland_Elevations)  
grid=Exdemo_Scotland_Elevations
```

1) VARIOGRAPHY

7. VARIOGRAM FOR GRID

```
data(Exdemo_Scotland_Elevations)
grid=Exdemo_Scotland_Elevations
plot(grid,scale=1,col=topo.colors(100),
      pos.legend=5)
```

1) VARIOGRAPHY

7. VARIOGRAM FOR GRID

```
data(Exdemo_Scotland_Elevations)
grid=Exdemo_Scotland_Elevations
plot(grid,scale=1,col=topo.colors(100),
      pos.legend=5)

vario=vario.grid(grid,nlag=30)
plot(vario)
```

1) VARIOGRAPHY

7. VARIOGRAM FOR GRID

```
model.auto(vario,struct=melem.name(c(1,3,2)))
```

1) VARIOGRAPHY

7. VARIOGRAM FOR GRID

```
model.auto(vario,struct=melem.name(c(1,3,2)))  
model.auto(vario,struct=melem.name(c(1,3,2)),  
            auth.aniso=F)
```

2) ESTIMATION

INTRODUCTION

- 1 Preamble
 - Load and prepare the data
 - Create the variogram model
 - Create a neighborhood
- 2 Perform kriging
- 3 Cross-validation
- 4 Moving neighborhood

2) ESTIMATION

PREAMBLE (1)

#1) Prepare the data

```
data(Exdemo_Scotland_Temperatures)
data(Exdemo_Scotland_Elevations)
dat=Exdemo_Scotland_Temperatures
grid=Exdemo_Scotland_Elevations
dat=db.locate(dat,4)
```

2) ESTIMATION

PREAMBLE (2)

#2) Define the model

```
vario=vario.calc(dat,lag=10,nlag=40)
```

```
model=model.auto(vario,
```

```
                struct=melem.name(c(1,3,2)))
```

```
# We could have used model.create (script) or
```

```
#           model.input (interface)
```

```
# to create a model.
```

2) ESTIMATION

PREAMBLE (3)

#3) Define the neighborhood

```
unique.neigh=neigh.init(type=0,ndim=2)
```

#Equivalent to

```
unique.neigh=neigh.input(ndim=2)
```

```
#answer 0
```

2) ESTIMATION

2. KRIGING

```
result0=kriging(dat,grid,model=model,  
                neigh=unique.neigh)
```

2) ESTIMATION

2. KRIGING

```
result0=kriging(dat,grid,model=model,  
                neigh=unique.neigh)  
  
plot(result0,scale=1,  
      col=topo.colors(100),pos.legend=5)  
plot(dat,add=T)
```

2) ESTIMATION

2. KRIGING

```
result0=kriging(dat,grid,model=model,  
                neigh=unique.neigh)  
  
plot(result0,scale=1,  
      col=topo.colors(100),pos.legend=5)  
plot(dat,add=T)  
  
plot(result0,name=7,scale=1,  
      col=topo.colors(100),pos.legend=5)  
plot(dat,add=T)
```

2) ESTIMATION

3. CROSS-VALIDATION

```
res.xv = xvalid(dat,model,neigh=unique.neigh)
hist(res.xv[,6],breaks=20)
hist(res.xv[,7],breaks=20)
mean(res.xv[,7]^2,na.rm=T)
plot(res.xv,scale=1)
```

2) ESTIMATION

3. CROSS-VALIDATION

```
res.xv = xvalid(dat,model,neigh=unique.neigh)
hist(res.xv[,6],breaks=20)
hist(res.xv[,7],breaks=20)
mean(res.xv[,7]^2,na.rm=T)
plot(res.xv,scale=1)

plot(result0,scale=1,
      col=topo.colors(100),pos.legend=5)
plot(res.xv,scale=1,add=T,
      col=1+as.numeric(res.xv[,6]>0))
```


2) ESTIMATION

4. MOVING NEIGHBORHOOD

```
moving.neigh=neigh.input()  
#answers 2, 5, 200, n, n, 50
```

2) ESTIMATION

4. MOVING NEIGHBORHOOD

```
moving.neigh=neigh.input()  
#answers 2, 5, 200, n, n, 50  
  
result1=kriging(dat,grid,  
                model=model,neigh=moving.neigh)
```

2) ESTIMATION

4. MOVING NEIGHBORHOOD

```
moving.neigh=neigh.input()
#answers 2, 5, 200, n, n, 50

result1=kriging(dat,grid,
                model=model,neigh=moving.neigh)
plot(result1,scale=1,
      col=topo.colors(100),pos.legend=5)
```

2) ESTIMATION

4. MOVING NEIGHBORHOOD

```
moving.neigh=neigh.input()
#answers 2, 5, 200, n, n, 50

result1=kriging(dat,grid,
                model=model,neigh=moving.neigh)
plot(result1,scale=1,
      col=topo.colors(100),pos.legend=5)
plot(result0[,6],result1[,6],cex=.2)
abline(0,1,col=2)
```

3) MULTIVARIATE GEOSTATISTICS

INTRODUCTION

- The temperature stations are not located at high elevations
- The temperature depends on the altitude
- We know the elevation of the stations (and some other elevations)
- We want to use this auxiliary information to improve the prediction
- In a second step, we will use the full Digital Terrain Model (DTM)

3) MULTIVARIATE GEOSTATISTICS

INTRODUCTION

- 1 Preamble
- 2 Exploratory data analysis (bivariate)
- 3 Cross-variogram model
- 4 Co-kriging
- 5 Collocated co-kriging
- 6 External drift

3) MULTIVARIATE GEOSTATISTICS

1. PREAMBLE

The results of ordinary kriging are stocked in `result0`.

3) MULTIVARIATE GEOSTATISTICS

1. PREAMBLE

The results of ordinary kriging are stocked in `result0`.

```
data(Exdemo_Scotland_Temperatures)
```

```
data(Exdemo_Scotland_Elevations)
```

```
dat=Exdemo_Scotland_Temperatures
```

```
grid=Exdemo_Scotland_Elevations
```


3) MULTIVARIATE GEOSTATISTICS

1. PREAMBLE

The results of ordinary kriging are stocked in `result0`.

```
data(Exdemo_Scotland_Temperatures)
```

```
data(Exdemo_Scotland_Elevations)
```

```
dat=Exdemo_Scotland_Temperatures
```

```
grid=Exdemo_Scotland_Elevations
```

Creation of the variable indicating if the temperature is available or if there is only the elevation.

```
dat=db.add(dat,
```

```
    "Available_Temp"=(!is.na(dat[,5])),
```

```
    loctype="NA")
```

3) MULTIVARIATE GEOSTATISTICS

1. PREAMBLE

The results of ordinary kriging are stocked in `result0`.

```
data(Exdemo_Scotland_Temperatures)
```

```
data(Exdemo_Scotland_Elevations)
```

```
dat=Exdemo_Scotland_Temperatures
```

```
grid=Exdemo_Scotland_Elevations
```

Creation of the variable indicating if the temperature is available or if there is only the elevation.

```
dat=db.add(dat,
```

```
    "Available_Temp"=(!is.na(dat[,5])),
```

```
    loctype="NA")
```

```
unique.neigh=neigh.init(type=0,ndim=2)
```

3) MULTIVARIATE GEOSTATISTICS

2. EXPLORATORY DATA ANALYSIS (BIVARIATE)

```
dat=db.locate(dat,6,"sel")  
plot(dat)  
db.stat(dat,fun="maxi",names=4)
```

3) MULTIVARIATE GEOSTATISTICS

2. EXPLORATORY DATA ANALYSIS (BIVARIATE)

```
dat=db.locate(dat,6,"sel")
plot(dat)
db.stat(dat,fun="maxi",names=4)
dat[,6]=!dat[,6]
plot(dat,add=T,col=3)
db.stat(dat,fun="maxi",names=4)
```

3) MULTIVARIATE GEOSTATISTICS

2. EXPLORATORY DATA ANALYSIS (BIVARIATE)

```
dat=db.locate(dat,6,"sel")
plot(dat)
db.stat(dat,fun="maxi",names=4)
dat[,6]=!dat[,6]
plot(dat,add=T,col=3)
db.stat(dat,fun="maxi",names=4)
dat[,6]=!dat[,6]
correlation(dat,icol1=4,icol2=5,col=c(2,3),
            pos.legend=1)
```

3) MULTIVARIATE GEOSTATISTICS

2. EXPLORATORY DATA ANALYSIS (BIVARIATE)

```
dat=db.locate(dat,6,"sel")
plot(dat)
db.stat(dat,fun="maxi",names=4)
dat[,6]=!dat[,6]
plot(dat,add=T,col=3)
db.stat(dat,fun="maxi",names=4)
dat[,6]=!dat[,6]
correlation(dat,icol1=4,icol2=5,col=c(2,3),
            pos.legend=1)
regression(dat,icol1=4,icol2=5,flag.draw=T,
           save.coeff= T)
```

3) MULTIVARIATE GEOSTATISTICS

3. CROSS-VARIOGRAM MODEL

```
cross_vario=vario.calc(dat,lag=10,nlag=40)
m_model=model.auto(cross_vario,
                   struct=melem.name(c(1,3,2)),
                   flag.noreduce=T)
```

3) MULTIVARIATE GEOSTATISTICS

4. CO-KRIGING (*isotopic*)

Isotopic

```
dat=db.locate(dat,6,"sel")
```


3) MULTIVARIATE GEOSTATISTICS

4. CO-KRIGING (*isotopic*)

Isotopic

```
dat=db.locate(dat,6,"sel")
```

```
resultm1=kriging(dat,grid,model=m_model,  
                 neigh=unique.neigh)
```

```
plot(resultm1,name="K.January_temp.estim",  
      scale=1,pos.legend=5,col=topo.colors(100))
```

3) MULTIVARIATE GEOSTATISTICS

4. CO-KRIGING (*heterotopic*)

Heterotopic

```
dat=db.locate(dat,6)
```

3) MULTIVARIATE GEOSTATISTICS

4. CO-KRIGING (*heterotopic*)

Heterotopic

```
dat=db.locate(dat,6)
```

```
resultm2=kriging(dat,grid,model=m_model,  
                 neigh=unique.neigh)
```

```
plot(resultm2,name="K. January_temp.estim",  
      scale=1,pos.legend=5,col=topo.colors(100))
```

3) MULTIVARIATE GEOSTATISTICS

4. CO-KRIGING (*collocated*)

```
moving.neigh=neigh.input(ndim=2)  
#Answers 2, 5, 20, n, n, 50
```

3) MULTIVARIATE GEOSTATISTICS

4. CO-KRIGING (*collocated*)

```
moving.neigh=neigh.input(ndim=2)
```

```
#Answers 2, 5, 20, n, n, 50
```

```
resultm3=kriging(dat,grid,model=m_model,  
                 neigh=moving.neigh,flag.colk=T)
```

3) MULTIVARIATE GEOSTATISTICS

4. CO-KRIGING (*collocated*)

```
moving.neigh=neigh.input(ndim=2)
#Answers 2, 5, 20, n, n, 50

resultm3=kriging(dat,grid,model=m_model,
                 neigh=moving.neigh,flag.colk=T)

plot(resultm3,name="K.January_temp.estim",
     scale=1,pos.legend=5,col=topo.colors(100))
```

3) MULTIVARIATE GEOSTATISTICS

5. KRIGING WITH EXTERNAL DRIFT

```
dat=db.locate(dat,4,"f")  
grid=db.locate(grid,4,"f")
```

3) MULTIVARIATE GEOSTATISTICS

5. KRIGING WITH EXTERNAL DRIFT

```
dat=db.locate(dat,4,"f")
grid=db.locate(grid,4,"f")

dat_r=regression(dat,flag.bivar=FALSE)
vario=vario.calc(dat_r,nlag=40,lag=10)
mres=model.auto(vario,struct=melem.name(c(5,3)),
                upper=c(NA,NA,NA,300))
```


3) MULTIVARIATE GEOSTATISTICS

5. KRIGING WITH EXTERNAL DRIFT

```
resultm4=kriging(dat,grid,model=mres,  
                 uc=c("1","f1"),  
                 neigh=unique.neigh)
```

3) MULTIVARIATE GEOSTATISTICS

5. KRIGING WITH EXTERNAL DRIFT

```
resultm4=kriging(dat,grid,model=mres,  
                uc=c("1","f1"),  
                neigh=unique.neigh)  
  
plot(resultm4,name="K. January_temp.estim",  
     scale=1,pos.legend=5,  
     col=topo.colors(100))
```

4) SIMULATIONS

FIRST STEPS

```
grid=db.create(nx=c(100,100))  
model=model.create(vartype= "Gaussian",range=30)
```

4) SIMULATIONS

FIRST STEPS

```
grid=db.create(nx=c(100,100))
model=model.create(vartype= "Gaussian",range=30)
grid = simtub(,grid,model=model,nbtuba=1,seed=0)
plot(grid,scale=1,col=topo.colors(100))
```

Try several times the last two commands.

4) SIMULATIONS

FIRST STEPS

```
grid=db.create(nx=c(100,100))  
model=model.create(vartype= "Gaussian",range=30)  
  
grid = simtub(,grid,model=model,nbtuba=1,seed=0)  
plot(grid,scale=1,col=topo.colors(100))
```

Try several times the last two commands.

Try with

```
nbtuba = 2  
and  
nbtuba = 10  
and  
nbtuba = 100  
and  
nbtuba = 1000
```

4) SIMULATIONS

STATISTICAL FLUCTUATIONS

A **stationary Gaussian** random function $Y = (Y(x), x \in \mathbb{R}^d)$ with

- mean m
- variance σ^2
- correlation function $\rho(h)$
- variogram $\gamma(h) = \sigma^2(1 - \rho(h))$

is simulated on a domain V .

$(y(x), x \in V)$ is the realization

4) SIMULATIONS

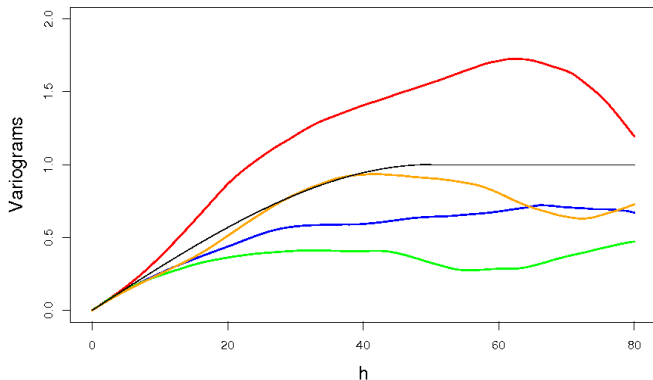
STATISTICAL FLUCTUATIONS

From the simulation $(y(x), x \in V)$ some **experimental** quantities can be computed:

- the mean $y_V = \frac{1}{|V|} \int_V y(x) dx$
- the dispersion variance $s^2(0|V) = \frac{1}{|V|} \int_V (y(x) - y_V)^2 dx$
- the regional variogram $\gamma_V(h) = \frac{1}{2|V \cap V_{-h}|} \int_{V \cap V_{-h}} (y(x) - y(x+h))^2 dx$
- some proportions $p(a) = \frac{1}{|V|} \int_V \mathbf{1}_{y(x) < a} dx$

4) SIMULATIONS

STATISTICAL FLUCTUATIONS

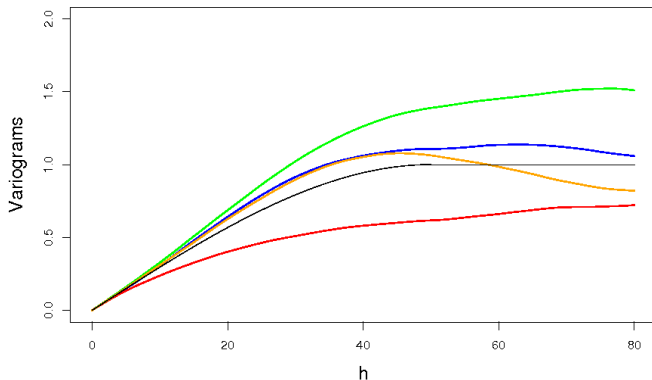


$$V = 100 \times 100$$

$$\gamma = Sph(1, 50)$$

4) SIMULATIONS

STATISTICAL FLUCTUATIONS

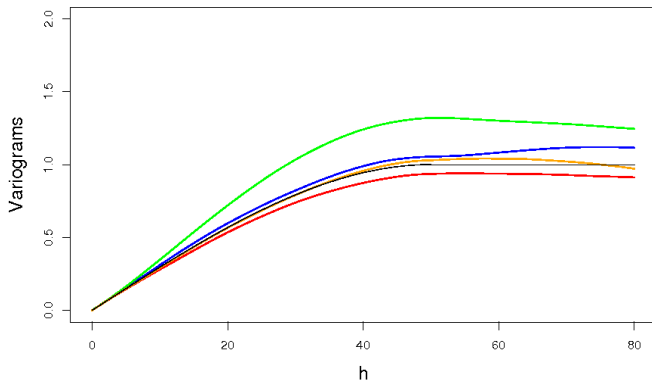


$V = 200 \times 200$

$\gamma = Sph(1, 50)$

4) SIMULATIONS

STATISTICAL FLUCTUATIONS

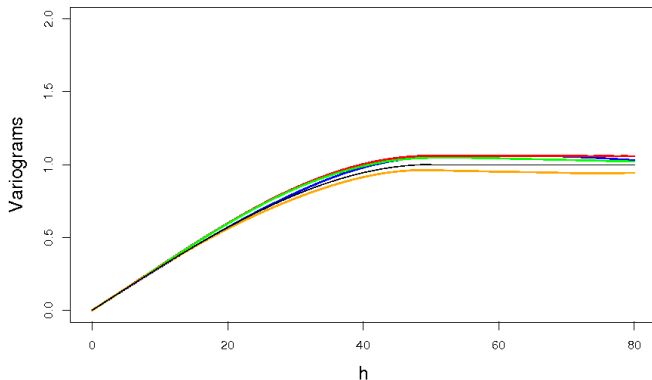


$V = 400 \times 400$

$\gamma = Sph(1, 50)$

4) SIMULATIONS

STATISTICAL FLUCTUATIONS



$$V = 800 \times 800$$

$$\gamma = Sph(1, 50)$$

4) SIMULATIONS

STATISTICAL FLUCTUATIONS

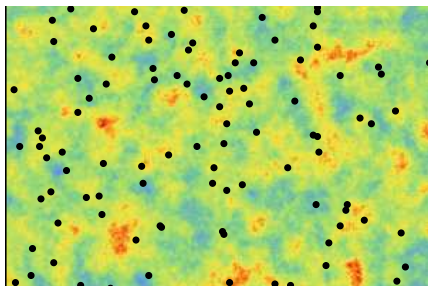
The experimental quantities **are not equal** to their corresponding quantities in the model:

- $y_V \neq m$
- $s^2(0|V) \neq \sigma^2$
- $\gamma_V(h) \neq \gamma(h)$
- $p(a) \neq G\left(\frac{a-m}{\sigma}\right)$ (G is the Gaussian cumulative distribution function)
- they differ from one simulation to another (**statistical fluctuations**)
- the experimental statistics tends to their corresponding quantities in the model when the size of V increases (ergodicity)

4) SIMULATIONS

CONDITIONAL SIMULATIONS

How to simulate a realization $\{y(x), x \in \mathbb{R}^d\}$ of a second order **Gaussian** random function $\{Y(x), x \in \mathbb{R}^d\}$ with mean m and covariance function C with respect to the data $\{Y(x_i) = y_i, i = 1, \dots, n\}$?



4) SIMULATIONS

PRINCIPE

Let

$$Y(x) = Y^{SK}(x) + Y(x) - Y^{SK}(x)$$

where

$$Y^{SK}(x) = m + \sum_{j=1}^n \lambda_j(x)[Y(x_j) - m] \quad \text{simple kriging}$$

$$Y(x) - Y^{SK}(x) \quad \text{kriging residuals}$$

Y^{SK} et $Y - Y^{SK}$ are to independant Gaussian random functions

4) SIMULATIONS

ALGORITHM

- (i) make an unconditional simulation $\{s(x), x \in \mathbb{R}^d\}$ and set $s_j = s(x_j)$.
- (ii) for all $x \in \mathbb{R}^d$, compute the kriging weights $(\lambda_j(x), j = 1, \dots, n)$
- (iii) set

$$\begin{aligned}y^{CS}(x) &= y^{SK}(x) + s(x) - s^{SK}(x) \\ &= s(x) + \sum_{j=1}^n \lambda_j(x)(y_j - s_j)\end{aligned}$$

4) SIMULATIONS

CHECKING

- If $x = x_i$, then $y^{CS}(x_i) = y_i + s(x_i) - s(x_i) = y_i$

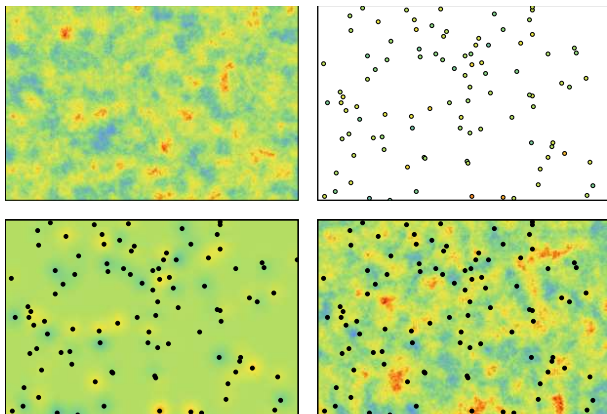
The conditioning data are honored

- If $C(x - x_\alpha) \simeq 0$ for all $i = 1, \dots, n$, then $y^{CS}(x) \simeq m + s(x) - m = s(x)$

For points distant from the data, the simulation is unconditional

4) SIMULATIONS

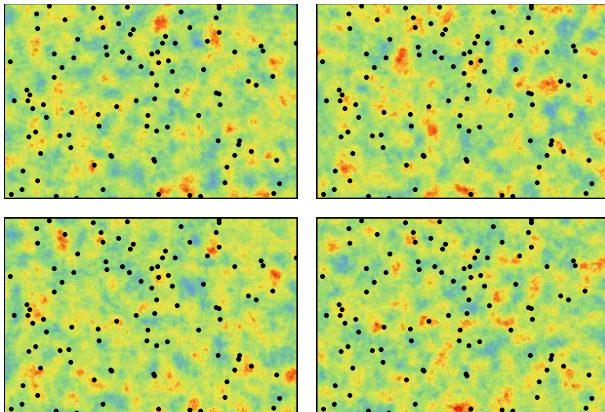
ILLUSTRATION



Simulation (TL), Synthetic data (TR), Simple kriging (BL), a conditional simulation (BR)

4) SIMULATIONS

4 CONDITIONAL SIMULATIONS



4) SIMULATIONS

GAUSSIAN TRANSFORM

- Limitation of the Gaussian model:
 - Symmetrical distribution
 - Inadapted to model data with heavy tail histogram
 - Conditional variance does not depend on the data (homoscedasticity)
- Anamorphosis: bijective function Φ defined on \mathbb{R}
- Model: data are a realization of a second order stationary Gaussian random function transformed by Φ
- Example : The **log-normal** model

$$\Phi(Y) = \exp(\mu + \sigma Y)$$

4) SIMULATIONS

CONDITIONAL SIMULATION OF A TRANSFORMED GAUSSIAN RANDOM FUNCTION

MODEL

$(z(x_1), \dots, z(x_n))$ is a realization of $(Z(x), x \in \mathbb{R}^d)$, a transformed Gaussian random function:

- $Z(x) = \Phi(Y(x))$ for all $x \in \mathbb{R}^d$
- $(Y(x), x \in \mathbb{R}^d)$ a second order stationary Gaussian random function with correlation function C

How to simulate a realization $(z(x), x \in \mathbb{R}^d)$ from $(Z(x), x \in \mathbb{R}^d)$ such as $Z(x_i) = z(x_i)$ for $i = 1, \dots, n$?

4) SIMULATIONS

ALGORITHM

(i) *transform the data in the Gaussian scale*

$$y_i = \Phi^{-1}(z(x_i))$$

(ii) *Perform a conditional simulation $\{y(x), x \in \mathbb{R}^d\}$ from the Gaussian random function $(Y(x), x \in \mathbb{R}^d)$ which honors the data in the Gaussian case*

$$Y(x_i) = y_i$$

(iii) *transform back the simulation for all $x \in \mathbb{R}^d$*

$$z(x) = \Phi(y(x))$$

4) SIMULATIONS

CONDITIONAL SIMULATIONS

```
data(Exdemo_Scotland_Temperatures)
data(Exdemo_Scotland_Elevations)
dat=Exdemo_Scotland_Temperatures
grid=Exdemo_Scotland_Elevations
dat=db.locate(dat,4)
vario=vario.calc(dat,lag=10,nlag=40)
model=model.auto(vario, flag.noreduce=T,
                 struct=melem.name(c(1,2,3)),
                 lower=c(0.02,NA,NA,NA,NA),
                 upper=c(0.02,NA,NA,NA,NA))
unique.neigh=neigh.init(type=0,ndim=2)
```

4) SIMULATIONS

CONDITIONAL SIMULATIONS

```
m=mean(dat[,5],na.rm=T)
simu = simtub(dat,grid,model=model,mean=m,
              neigh=unique.neigh,uc="",
              nbtuba=1000,nbsimu=100)
plot(simu,scale=1,col=topo.colors(100),
      name=6,zlim=c(-2,5),pos.legend=5)
```

4) SIMULATIONS

CONDITIONAL SIMULATIONS

```
m=mean(dat[,5],na.rm=T)
simu = simtub(dat,grid,model=model,mean=m,
              neigh=unique.neigh,uc="",
              nbtuba=1000,nbsimu=100)
plot(simu,scale=1,col=topo.colors(100),
      name=6,zlim=c(-2,5),pos.legend=5)
result=db.compare(simu,fun="mean",name=6:105)
```


4) SIMULATIONS

COMPARISON WITH SIMPLE KRIGING

```
result0=kriging(dat,grid,uc="",mean=m,  
                model=model,neigh=unique.neigh)
```

4) SIMULATIONS

COMPARISON WITH SIMPLE KRIGING

```
result0=kriging(dat,grid,uc="",mean=m,  
                model=model,neigh=unique.neigh)  
  
plot(result[,106],result0[,6],cex=.2)  
abline(0,1,col=2)
```

4) SIMULATIONS

COMPARISON WITH SIMPLE KRIGING

```
result0=kriging(dat,grid,uc="",mean=m,  
                model=model,neigh=unique.neigh)
```

```
plot(result[,106],result0[,6],cex=.2)  
abline(0,1,col=2)
```

```
result=db.compare(simu,fun="stdv",name=6:105)  
plot(result[,107],result0[,7],cex=.2)  
abline(0,1,col=2)
```

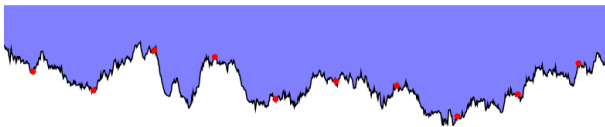
4) SIMULATIONS

OPTIMAL PREDICTOR

Example: a submarine cable has to be set on the seabed between Lisbonne and New-York.

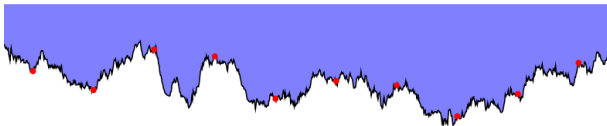
How to predict its length from the bathymetry z sampled every 200m.

Uncertainty?



$$l = \int_a^b \sqrt{1 + [z(x)']^2} dx$$

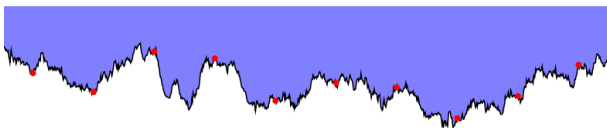
NATURAL IDEA



Measuring the length of the predicted (kriged) bathymetry \hat{z}

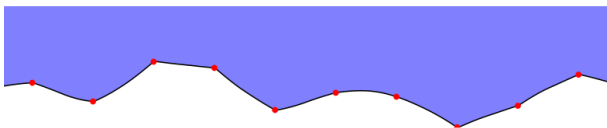
$$\hat{l} = \int_a^b \sqrt{1 + [\hat{z}(x)']^2} dx$$

NATURAL IDEA



Measuring the length of the predicted (kriged) bathymetry \hat{z}

$$\hat{l} = \int_a^b \sqrt{1 + [\hat{z}(x)']^2} dx$$



Systematic under-estimation: predicted trajectory is much smoother than the actual one

4) SIMULATIONS

OPTIMAL PREDICTOR

- (X_0, X_1, \dots, X_n) is a random vector with finite variance.
- The **regression** of X_0 on (X_1, \dots, X_n) is the function r such as the mean squared error

$$E[(X_0 - r(X_1, \dots, X_n))^2]$$

is minimal

4) SIMULATIONS

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is minimal

- It is given by the conditional expectation of X_0 knowing X_1, \dots, X_n :

$$r(x_1, \dots, x_n) = E[X_0 | X_1 = x_1, \dots, X_n = x_n].$$

4) SIMULATIONS

OPTIMAL PREDICTOR

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- It is given by the conditional expectation of X_0 knowing X_1, \dots, X_n :

$$r(x_1, \dots, x_n) = E[X_0 | X_1 = x_1, \dots, X_n = x_n].$$

- In the multi-Gaussian case, the regression is **linear**
In other words, simple kriging is **optimal**

4) SIMULATIONS

OPTIMAL PREDICTOR

- In other cases, we can compute $r(x_1, \dots, x_n)$ by averaging conditional simulations of X_0 knowing $X_1 = x_1, \dots, X_n = x_n$:

$$E[X_0 | X_1 = x_1, \dots, X_n = x_n] \simeq \frac{1}{N} \sum_{i=1}^N X_0^{(i)}$$

where $X_0^{(i)}$ is the i^{th} conditional simulation of X_0 knowing $X_1 = x_1, \dots, X_n = x_n$

4) SIMULATIONS

OPTIMAL PREDICTOR

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$$E[X_0 | X_1 = x_1, \dots, X_n = x_n] \simeq \frac{1}{N} \sum_{i=1}^N X_0^{(i)}$$

where $X_0^{(i)}$ is the i^{th} conditional simulation of X_0 knowing $X_1 = x_1, \dots, X_n = x_n$

- For the submarine cable $X_0 \equiv \text{Length}$.

4) SIMULATIONS

PREAMBLE

```
data(Exdemo_bathymetry_1D)
dat=Exdemo_bathymetry_1D

v=vario.calc(dat,lag=1,nlag=50)
model=model.auto(v,melem.name(c(5,2)),
                 flag.noreduce=T)

neigh=neigh.init(ndim=1,type=0)
```

4) SIMULATIONS

FUNCTION TO COMPUTE THE LENGTH OF THE CABLE

```
calcul_length=function(db,index=1){  
  x=db.extract(db,"x1")  
  z=db.extract(db,paste("z",index,sep=""))  
  sum(sqrt((x[-1]-x[-length(x)])^2  
    +(z[-1]-z[-length(z)])^2))  
}
```

4) SIMULATIONS

LENGTH OF THE KRIGING

```
db_res=db.create(nx=10000,dx=1)
db_krig=kriging(dat,db_res,model=model,
                neigh=neigh)
resk=calcul_length(db_krig,1)
```

4) SIMULATIONS

CONDITIONAL SIMULATIONS

```
Nbsimus=100
```

```
db_simu=simtub(dat,db_res,model=model,  
              mean=-1400,neigh=neigh,uc="",  
              nbtuba=1000,nbsimu=Nbsimus)
```

4) SIMULATIONS

CONDITIONAL SIMULATIONS

```
Nbsimus=100
db_simu=simtub(dat,db_res,model=model,
               mean=-1400,neigh=neigh,uc="",
               nbtuba=1000,nbsimu=Nbsimus)

res=rep(NA,Nbsimus)
for(i in 1:Nbsimus)
  res[i]=calcul_length(db_simu,i)
```


4) SIMULATIONS

DISPLAY THE RESULTS

```
hist(res,prob=T,xlim=range(c(resk,res)))  
abline(v= calcul_length(db_krig),col=3)  
abline(v= mean(res),col=4)
```

4) SIMULATIONS

DISPLAY THE RESULTS

```
hist(res,prob=T,xlim=range(c(resk,res)))  
abline(v= calcul_length(db_krig),col=3)  
abline(v= mean(res),col=4)  
  
plot(dat,xlim=c(0,1000),type="p",cex=.5,col=2)  
plot(db_krig,add=T,lwd=2,col=3)  
plot(db_simu,add=T,col=4,name=3,lty=2)
```

4) SIMULATIONS

COMPARE WITH THE REALITY

```
data(Exdemo_bathymetry_1D_full)
grid=Exdemo_bathymetry_1D_full
plot(grid,add=T,col=2)
```

4) SIMULATIONS

COMPARE WITH THE REALITY

```
data(Exdemo_bathymetry_1D_full)
grid=Exdemo_bathymetry_1D_full
plot(grid,add=T,col=2)

true_length=calcul_length(grid)
```

4) SIMULATIONS

COMPARE WITH THE REALITY

```
data(Exdemo_bathymetry_1D_full)
grid=Exdemo_bathymetry_1D_full
plot(grid,add=T,col=2)

true_length=calcul_length(grid)

hist(res,prob=T,xlim=range(c(resk,res)))
abline(v= calcul_length(db_krig),col=3)
abline(v= mean(res),col=4)
abline(v=true_length,col=2)
```